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L. Grigoryeva, PhD, Assoc. Prof.
Kyiv National University of Construction and Architecture, 31 Povitroflotskiy Ave., Kiev, Ukraine 03680; e-mail: l_grigoryeva@ukr.net

CALCULATION OF CYLINDRICAL MULTILAYER ELECTROMECHANICAL TRANSDUCER AT DIFFERENT POLARIZATION TYPES IN NON-STATIONARY MODES

Л.О. Григор'єва. **Розрахунок багатошарового електромеханічного перетворювача циліндричної форми при різних типах поляризації в нестаціонарних режимах роботи.** Розвинуто чисельний спосіб дослідження коливань циліндричних багатошарових електромеханічних перетворювачів з електродованими поверхнями спряження при електричних збуреннях. Проводиться дослідження параметрів електромеханічного стану перетворювача в динаміці в залежності від кількості електродованих шарів та напрямку поляризації. Встановлено залежність періодичності радіальних коливань від геометричних розмірів та пропорційність між амплітудними значеннями переміщень та напружень циліндрів з зустрічно поляризованими шарами та кількістю шарів. Досліджено коливання зовнішньої поверхні та їх періодичність при збільшенні внутрішнього отвору циліндра.
Ключові слова: п'єзокерамічний перетворювач, нестаціонарні коливання, електричне збурення, багатошаровий п'єзоелемент, напрямок поляризації

L. Grigoryeva. **Calculation of cylindrical multilayer electromechanical transducer at different polarization types in non-stationary modes.** Numerical research method for oscillations of cylindrical multilayer electromechanical transducers with electrode conjugation surfaces at electrical disturbances is developed. Electromechanical state parameters of transducer depending on the number of electrode layers and polarization direction are investigated dynamically. Dependence of the radial oscillations frequency on geometric dimensions and proportionality between amplitude values of displacements and stresses in cylinders with oncoming polarization layers and the number of layers are established. Correlation between outer surface oscillations and cylinder inner opening size is studied.
Keywords: piezoceramic transducer, non-stationary oscillations, electrical disturbances, multilayer piezoelectric element, polarization direction

Introduction. Piezoceramic electromechanical multilayer cylindrical transducers are often used in acoustoelectric devices for receiving and emitting an acoustic signal including non-stationary one. In this regard it is necessary to study the transmission of electroelastic disturbances in multilayer piezoceramic bodies of cylindrical shape under various manufacturing parameters and operating conditions.

In this work oscillations of a multilayer piezoceramic hollow cylinder with electrode surfaces of separation of layers are explored. Oscillations are caused by sudden potential difference, alternating-sign for the adjacent layers. An important issue in designing such bodies is the choice of polarization direction of layers, which affects the emissive ability of the element. Therefore the study of electromechanical state of the piezoelectric element in different polarization types and the choice of optimal variant is accepted as the purpose of the work.

Analysis of recent research and publications. Research of oscillations of piezoceramic elements of constructions with non-stationary electrical and mechanical loads is an important aspect for the mechanics of conjugated fields. The fundamental issues of electroelasticity problems formulation,

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including in a cylindrical coordinate system [1, etc.], are considered. Analytical solutions of a number of important problems of hydroacoustics in non-stationary formulation [2] are proposed. The classical approaches to the formulation and solving of axisymmetric oscillations of multilayer massive bodies and shells of cylindrical and spherical form are described [3, 4, etc.]. Oscillations of homogeneous radially polarized cylinders with non-stationary loads [5, 6] were reflected. The stationary and non-stationary oscillations of piezoceramic bodies with curved surfaces were investigated [7, 8]. Harmonic oscillations of heterogeneous magneto-electroelastic cylinders were studied [9]. Axisymmetric oscillations in cylinders with piezoceramic radially polarized layers were studied [10].

The publications analysis shows that the study of the disturbance transmission in electroelastic multilayer cylinders (and other rotation bodies that are planned for research in subsequent works) depending on the number of layers and their polarization direction as well as the conditions of securing and loading is indeed an urgent and important task in the design process of devices that operate based on piezoelectric effect.

Problem statement. The radial oscillations of a multilayer hollow cylinder consisting of n piezoceramic polarized through-thickness layers with separating surfaces $R_0 < ... < R_k < ... < R_n$, where R_n – external, R_0 – internal radius of a cylinder are studied. Oscillations of a k layer are described by the motion equation and the electric induction equation in a cylindrical coordinate system

$$\frac{\partial \sigma^k{}_{rr}}{\partial r} + \frac{\sigma^k{}_{rr} - \sigma^k{}_{\theta\theta}}{r} = \rho^k \frac{\partial^2 u^k}{\partial t^2}, \tag{1}$$

$$\frac{\partial D^k{}_r}{\partial r} + \frac{D^k{}_r}{r} = 0, \tag{2}$$

under material ratio in layers with radial polarization direction ($i = 1...n$)

$$\sigma^k{}_{rr} = c_{33}{}^k \frac{\partial u^k}{\partial r} + c_{13}{}^k \frac{u^k}{r} + e_{33}{}^k \frac{\partial \varphi^k}{\partial r};$$

$$\sigma^k{}_{\theta\theta} = c_{13}{}^k \frac{\partial u^k}{\partial r} + c_{11}{}^k \frac{u^k}{r} + e_{31}{}^k \frac{\partial \varphi^k}{\partial r};$$

$$D^k{}_r = e_{33}{}^k \frac{\partial u^k}{\partial r} + e_{31}{}^k \frac{u^k}{r} - \varepsilon_{33}{}^k \frac{\partial \varphi^k}{\partial r}.$$

In the case of an alternating polarization direction we have

$$\begin{gathered}\sigma^k{}_{rr} = c_{33}{}^k \frac{\partial u^k}{\partial r} + c_{13}{}^k \frac{u^k}{r} + (-1)^k e_{33}{}^k \frac{\partial \varphi^k}{\partial r}; \\ \sigma^k{}_{\theta\theta} = c_{13}{}^k \frac{\partial u^k}{\partial r} + c_{11}{}^k \frac{u^k}{r} + (-1)^k e_{31}{}^k \frac{\partial \varphi^k}{\partial r}; \\ D^k{}_r = (-1)^k e_{33}{}^k \frac{\partial u^k}{\partial r} + (-1)^k e_{31}{}^k \frac{u^k}{r} - \varepsilon_{33}{}^k \frac{\partial \varphi^k}{\partial r}.\end{gathered} \tag{3}$$

Here $u^k(r,\ t)$ and $\sigma^k{}_{ij}$ – mechanical displacement and stress, $D_r^k(r,t)$ – component of radial direction vector of electrical induction; $\varphi^k(r,\ t)$ – electrical potential; ρ^k – material density; $c_{ij}{}^k$ – elasticity module at a constant electric field; ε_{33}^k – dielectric capacity at constant deformation; e_{ij}^k – piezoelectric module of a k layer.

Initial conditions are imposed on displacement and its speed

$$u^k(r,\ 0) = \overset{0}{u}{}^k(r); \quad \frac{\partial u^k}{\partial t}(r,\ t=0) = \overset{1}{u}{}^k(r). \tag{4}$$

Stress or displacement is set on the outer surfaces

$$u(R_a,\ t) = U_a(t) \ \wedge \ \sigma_r(R_a,\ t) = p_a(t), \quad a = 0,\ n. \tag{5}$$

The conditions of full contact are met between the layers $k = 1, ..., n-1$

$$u^k(R_k) = u^{k+1}(R_k), \quad \sigma_{rr}^k(R_k) = \sigma_x^{k+1}(R_k), \quad \varphi^k(R_k) = \varphi^{k+1}(R_k). \tag{6}$$

The electrodes in layers separating surfaces and outer surfaces indicate the potential difference

$$\varphi(R_k) = (-1)^{k+1} V(t), \quad k = 0, \ ..., \ n. \tag{7}$$

In [10] on the layers separating surfaces of the piezoceramic sphere, in addition to the condition of electric potential equality, the electrical induction continuity conditions are given by analogy with (6). In our case this condition is not necessary because the electrical induction continuity condition is placed if internal electrodes are absent or not connected to electric field.

For the universality of the solution we introduce dimensionless variables

$$\bar{r} = \frac{r}{l_0}; \quad \bar{t} = \frac{t}{t_l}; \quad \bar{u} = \frac{u}{l_0}; \quad \bar{\sigma} = \frac{\sigma}{c_{00}}; \quad \bar{\varphi} = \varphi\sqrt{\frac{\varepsilon_{00}}{c_{00} l_0^2}}; \quad \bar{D} = \frac{D}{\sqrt{c_{00}\varepsilon_{00}}};$$

$$\bar{\rho} = \frac{\rho}{\rho_{00}}; \quad \bar{c}_{ij} = \frac{c_{ij}^E}{c_{00}}; \quad \bar{e}_{ij} = \frac{e_{ij}}{\sqrt{c_{00}\varepsilon_{00}}}; \quad \bar{\varepsilon}_{33} = \frac{\varepsilon_{33}^S}{\varepsilon_{00}}, \tag{8}$$

where l_0, ρ_{00}, c_{00}, ε_{00}, $t_l = l_0\sqrt{\rho_{00} / c_{00}}$ – standardized values. With such nondimensionalization the output equations will not change.

In the transition in (1) (2) towards displacements and electric potential being the solving functions (3) for k -layer we receive

$$\rho\frac{\partial^2 u^k}{\partial t^2} = c_{33}^k\frac{\partial^2 u^k}{\partial r^2} + \frac{c_{33}^k}{r}\frac{\partial u^k}{\partial r} - c_{11}^k\frac{u^k}{r^2} + e_{33}^k\frac{\partial^2\varphi^k}{\partial r^2} + \frac{e_{33}^k - e_{13}^k}{r}\frac{\partial\varphi^k}{\partial r}; \tag{9}$$

$$e_{33}^k\frac{\partial^2 u^k}{\partial r^2} + \frac{e_{13}^k + e_{33}^k}{r}\frac{\partial u^k}{\partial r} - \varepsilon_{33}^k\left(\frac{\partial^2\varphi^k}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi^k}{\partial r}\right) = 0. \tag{10}$$

Hereinafter in order to take into account the polarization direction in layers we consider $e_{33}^k = e_{33}$ in the case of polarization of all layers in the radial direction, and $e_{33}^k = (-1)^{k-1} e_{33}$ if the first layer is polarized radially, the further ones are polarized alternating-sign next but one.

Thus we obtain the initial-boundary value problem, which is described by the equation (9) with initial and boundary conditions (4) (5). The electric potential of each time layer is determined by displacement from (10) under the conditions (7).

Numerical solution. In order to solve the initial-boundary value problem we construct a numerical scheme based on difference approximations. Within the integration interval $R_0 < ... < R_k < ... < R_n$ we introduce the partition:

$$\Omega = \{r_{m(k-1)+i} = R_{k-1} + (i-1)\Delta_k, \ \ \Delta_k = (R_k - R_{k-1}) / m, \ k = 1, ..., n, \ i = 1, ..., m+1\}.$$

Let us proceed from the continuum functions to the desired discrete values of functions at the partition points: $u(r_{m(k-1)+i}) = u_{m(k-1)+i-1}$, $\varphi(r_{m(k-1)+i}) = \varphi_{m(k-1)+i-1}$.

The difference form of the equations (9), (10) that is formulated for internal partition points takes the following form:

$$\begin{aligned}\rho\frac{\partial^2 u_{m(k-1)+i}}{\partial t^2} &= \left(\frac{c_{33}^k}{\Delta_k^2} - \frac{c_{33}^k}{\Delta_k r_{m(k-1)+i}}\right) u_{m(k-1)+i-1} + \left(-\frac{c_{11}^k}{r_{m(k-1)+i}^2} - 2\frac{c_{33}^k}{\Delta_k^2}\right) u_{m(k-1)+i} + \\ &+ \left(\frac{c_{33}^k}{\Delta_k^2} - \frac{c_{33}^k}{\Delta_k r_{m(k-1)+i}}\right) u_{m(k-1)+i+1} + \left(\frac{e_{33}^k}{\Delta_k^2} - \frac{(e_{33}^k - e_{13}^k)}{\Delta_k r_{m(k-1)+i}}\right)\varphi_{m(k-1)+i-1} - \\ &- 2\frac{e_{33}^k}{\Delta_k^2}\varphi_{m(k-1)+i} + \left(\frac{e_{33}^k}{\Delta_k^2} - \frac{(e_{33}^k - e_{13}^k)}{\Delta_k r_{m(k-1)+i}}\right)\varphi_{m(k-1)+i+1};\end{aligned} \tag{11}$$

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$$\left(e_{33}^k-\frac{\Delta_k(e_{33}^k+e_{13}^k)}{r_{m(k-1)+i}}\right)u_{m(k-1)+i-1}-2e_{33}^k u_{m(k-1)+i}+\left(e_{33}^k+\frac{\Delta_k(e_{33}^k+e_{13}^k)}{r_{m(k-1)+i}}\right)u_{m(k-1)+i+1}=$$
$$=\left(\varepsilon_{33}^k-\frac{\Delta_k\varepsilon_{33}^k}{r_{m(k-1)+i}}\right)\varphi_{m(k-1)+i-1}-2\varepsilon_{33}^k\varphi_{m(k-1)+i}+\left(\varepsilon_{33}^k+\frac{\Delta_k\varepsilon_{33}^k}{r_{m(k-1)+i}}\right)\varphi_{m(k-1)+i+1}. \quad (12)$$

Electric potential on the outer surfaces and separating surfaces equals (7)

$$\varphi_{m(k-1)+1}=(-1)^k V(t), \qquad k=1,...,n+1. \quad (13)$$

Displacement on the outer surfaces is found from the difference form of material ratios, written with one-sided difference expressions with second-order accuracy

$$\sigma_{rr}(R_0)=\frac{c_{33}^1}{2\Delta_1}(-3u_1+4u_2-u_3)+\frac{c_{13}^1}{R_0}u_1+\frac{e_{33}^1}{2\Delta_1}(-3\varphi_1+4\varphi_2-\varphi_3)=p_0(t);$$

$$\sigma_{rr}(R_n)=\frac{c_{33}^n}{2\Delta_n}(3u_{nm+1}-4u_{nm}+u_{nm-1})+\frac{c_{13}^n}{R_n}u_{nm+1}+\frac{e_{33}^1}{2\Delta_n}(3\varphi_{nm+1}-4\varphi_{nm}+\varphi_{nm-1})=p_n(t);$$

$$u_1=\left(p_0(t)-\frac{c_{33}^1}{2\Delta_1}(4u_2-u_3)-\frac{e_{33}^1}{2\Delta_1}(3V(t)+4\varphi_2-\varphi_3)\right)/\left(\frac{c_{13}^1}{R_0}-3\frac{c_{33}^1}{2\Delta_1}\right);$$

$$u_{nm+1}=\left(p_n(t)-\frac{c_{33}^n}{2\Delta_n}(-4u_{nm}+u_{nm-1})-\frac{e_{33}^1}{2\Delta_n}(3(-1)^{n+1}V(t)-4\varphi_{nm}+\varphi_{nm-1})\right)/\left(\frac{3c_{33}^n}{2\Delta_n}+\frac{c_{13}^n}{R_n}\right). \quad (14)$$

Matching condition (6) converts to

$$\frac{c_{33}^k}{2\Delta_k}(3u_{mk+1}-4u_{mk}+u_{mk-1})+\frac{c_{13}^k}{R_k}u_{mk+1}+\frac{e_{33}^k}{2\Delta_k}(3\varphi_{mk+1}-4\varphi_{mk}+\varphi_{mk-1})=$$
$$=\frac{c_{33}^{k+1}}{2\Delta_{k+1}}(-3u_{mk+1}+4u_{mk+2}-u_{mk+3})+\frac{c_{13}^{k+1}}{R_k}u_{mk+1}+\frac{e_{33}^{k+1}}{2\Delta_{k+1}}(-3\varphi_{mk+1}+4\varphi_{mk+2}-\varphi_{mk+3}).$$

The displacement on separating surfaces is determined by displacement and electric potential at inner points:

$$u_{mk+1}=\left(\frac{c_{33}^k}{2\Delta_k}(4u_{mk}-u_{mk-1})+\frac{e_{33}^k}{2\Delta_k}(4\varphi_{mk}-\varphi_{mk-1})+\frac{c_{33}^{k+1}}{2\Delta_{k+1}}(4u_{mk+2}-u_{mk+3})+\right.$$
$$\left.+\frac{e_{33}^{k+1}}{2\Delta_{k+1}}(4\varphi_{mk+2}-\varphi_{mk+3})-\left(3\frac{e_{33}^k}{2\Delta_k}+3\frac{e_{33}^{k+1}}{2\Delta_{k+1}}\right)(-1)^{k+1}V(t)\right)/\left(\frac{c_{13}^k}{R_k}+3\frac{c_{33}^k}{2\Delta_k}-\frac{c_{13}^{k+1}}{R_k}+3\frac{c_{33}^{k+1}}{2\Delta_{k+1}}\right). \quad (15)$$

In matrix form the system of equations (11) (12), taking into account (13) – (15) at the inner partition points, turns into

$$\rho\frac{d^2U}{dt^2}=AU+B\Phi+F_1, \quad D\Phi=CU+F_2, \quad (16)$$

where $U=\{u_{m(k-1)+i-1}\}$, $\Phi=\{\varphi_{m(k-1)+i-1}\}$, $k=1,...,n$, $i=2,...,m$.

The system (16) time integration is done by introducing partition ω_t of the time interval $t\in[0, T]$ with the step Δt. The solution is searched by means of the explicit difference scheme

$$\ddot{u}^{p+1}=\frac{u^{p+1}-2u^p+u^{p-1}}{\Delta t^2},$$

in which the inner points displacement in the $p+1$ time layer is determinated from (11) because of the known displacement and electric potential of the p-time-layer, and the electric potential is found from (12) through inner displacements on the same layer. On the boundaries and surfaces of conjugation the displacement is determined from (14) (15).

When applying an implicit difference scheme (Newmark algorithm)

$$\ddot{u}^{p+1} = \frac{\dot{u}^{p+1} - \dot{u}^{p}}{\xi\Delta t} - \frac{1-\xi}{\xi}\ddot{u}^{p}, \quad \dot{u}^{p+1} = \frac{u^{p+1} - u^{p}}{\xi\Delta t} - \frac{1-\xi}{\xi}\dot{u}^{p},$$

where ξ – scheme parameter, system (14) forms a single matrix, from which all unknowns at inner points are determined at the same time. An important issue of explicit scheme application is partition step size selection in spatial and temporal coordinates, since the explicit scheme stability condition must be satisfied. Practice shows that it's usually enough to accept $\Delta_x \geq 10\Delta t$. For an implicit scheme steps of one order are chosen.

Numerical results. Let us consider the non-stationary oscillations of the ceramic ball PZT-4 [6] under zero initial conditions. Disturbance is given in the form of $V(t) = V_0 H(t)$, where $H(t)$ – Heaviside function. Such load is the most convenient for studying dependencies of transmission and reflection of disturbance waves. Oscillations research of *n*-layer cylinders with parallel and reverse polarization directions of layers is conducted. Geometric parameters are $R_0 / R_n = 0.5$, where n – the number of layers. The results are presented in dimensionless form.

Fig. 1 shows that in radially polarized cylinders with a pair and odd number of layers oscillations arise with the amplitude of one order, respectively. Oscillations represent a superposition of radial and thickness oscillations. The frequency of radial oscillations depends on the geometric dimensions of the cylinder [5] and is the same for all cases considered. As the number of cylinder layers grows the frequency of the thickness oscillations increases, which is associated with a decrease in the characteristic oscillatory size, in this case the layer thickness *h*. Maximum displacements arise in cylinders with an odd number of layers, which is explained by the summation of displacements with different signs occurring in adjacent layers. Polarization of such elements is carried out by applying positive potential difference to outer electrodes, the inner ones are not connected to the electric circuit. Maximum displacements in all cases arise on the inner surface, on the outer surface in the case of an odd number of layers we have $u_{\max}(R_n) \approx 2.5$ and $u_{\max}(R_n) \approx 0.8$ for a pair number of layers.

Fig. 2 illustrates the oscillations of the outer surface of the cylinder with the oncoming polarization of the layers. We see that the displacement is practically proportional to the number of layers, since the deformation of each layer is the same and is summed up with each other. The period of radial oscillations is the same as in the previous case (Fig. 1). The maximum displacement values are proportional to the maximum displacements of the odd number of layers under radial polarization: $u_{\max}(R_n) \approx 2.5n$.

The analysis of the results shows that the circumferential stress is also proportional to the number of layers. For radial stresses the dependence is not so linear, but stress also increases as the number of layers grows.

Fig. 3 depicts curves of radial stresses on the median surface and circumferential stresses on the outer surfaces. It can be seen that the maximum values for radial and circumferential stresses are approximately the

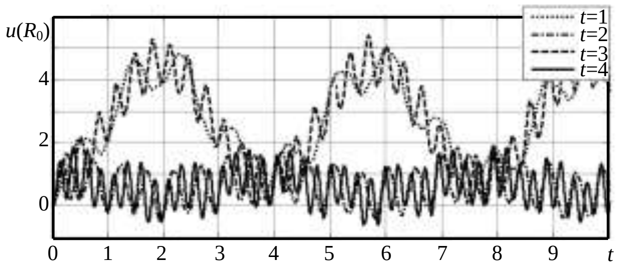

Fig. 1. Displacement of inner surface $r = R_0$ of n–layer cylinders with radial polarization of layers

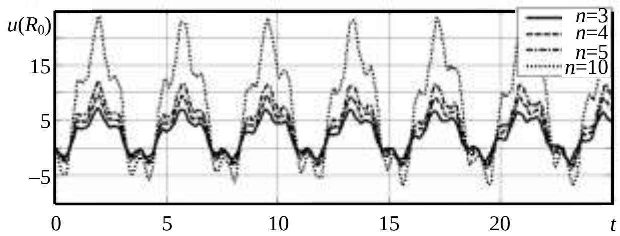

Fig. 2. Displacement of outer surface $r = R_n$ of n–layer cylinders with oncoming polarization of layers

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same, but they occur at different points of the cylinder. The frequency of change in radial stresses corresponds to the thickness oscillations of the cylinder [5] and the circumferential stresses change with the frequency corresponding to the radial oscillations.

An interesting research issue is the pattern of disturbance transmission in a cylinder. Let us consider the distribution of stresses at different moments of time ($n = 5$, oncoming polarization) using the results shown in Fig. 4.

Fig. 4 clearly shows that since the electric load application the stress inside the body attained certain values that are practically identical at all points of the layers (in this case $R_k = 0.6, 0.7, 0.8, 0.9$), but from the outer free surfaces there is wave transmission displacement with the speed $a_k = \sqrt{c_{33}^k / \rho^k}$ and, according to the graph, when $t > 0.2$ waves overlay.

Finally let us analyze the dependence of oscillations on the geometrical proportions in cylinders with the same number of layers. Fig. 5 shows the displacements that arise in five-layer cylinders with different inner openings. As the inner opening grows the displacement and oscillations frequency increase significantly. Displacements of the sphere with an opening $R_0 = 0.3R_n$ are more than five times less than displacements of the sphere $R_0 = 0.7R_n$.

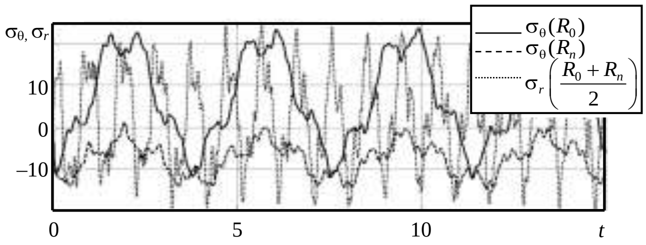

Fig. *3. Circumferential and radial stress of five-layer cylinders with oncoming polarization of layers*

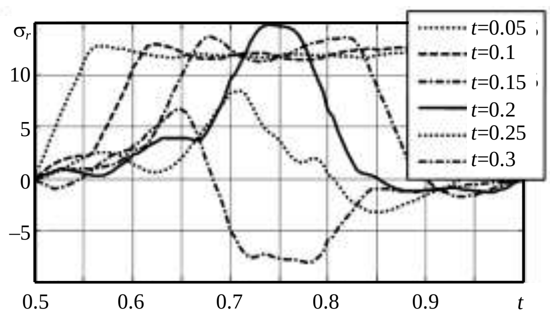

Fig. 4. Distribution of radial stress of five-layer cylinders radially

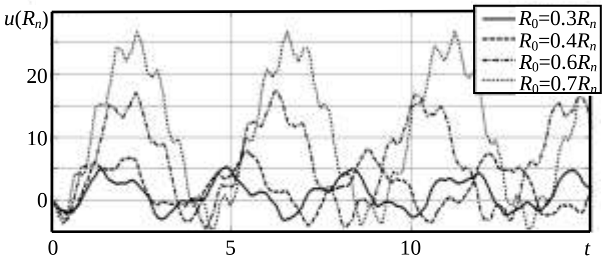

Fig. 5. Outer surface oscillations of five-layer cylinders of various radii

Conclusions. The developed approach enables effective research of dynamic electromechanical state of piezoceramic multilayer cylinders at electrical and mechanical loads. It is established that radially polarized cylinders with a pair number of layers emit high-frequency oscillations of low amplitude. The oscillations of radially polarized cylinders with an odd number of layers are close to the solid cylinder oscillations in amplitude and period.

The frequency of radial vibrations of multilayer cylinders coincides with the frequency of solid cylinders. The amplitude values of displacements of cylinders with oncoming polarized layers are proportional to the number of layers. With inner opening growth the outer surface oscillations and their frequency increase significantly.

Thus when designing electromechanical transducers operating in non-stationary modes it is necessary to take into account the above-mentioned regularities and carry out a detailed calculation of specific elements of structures. The proposed numerical scheme makes it possible to extend the scope of the method by changing the securing and loading conditions, materials of layers, including elastic ones.

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Григор’єва Людмила Олександрівна; Grigoryeva Ludmila, ORCID: https://orcid.org/0000-0001-7013-0327

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