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AUTOMATION OF ARCHES CALCULATION

В.Ф. Оробей, О.Ф. Дащенко, О.М. Лимаренко. **Автоматизація розрахунку арок.** Викладено процедуру автоматизації розрахунку напружено-деформованого стану кругових арок при обліку деформацій вигину і розтягування-стиснення, зосереджених і розподілених зовнішніх навантажень. Метою роботи є практичне застосування можливостей методу граничних елементів для вирішення досить трудомістких задач про напружено-деформований стан кругових арок і аркових конструкцій. Для досягнення поставленої мети виконано статичний розрахунок напружено-деформованого стану кругових арок в середовищі MATLAB. Для цього складається і вирішується система диференціальних рівнянь плоского деформування кругового стержня з урахуванням деформацій вигину і розтягування щодо радіального і тангенціального переміщень. В результаті розрахунку прийшли до висновку, що численні завдання розрахунку кілець і кільцевих систем можуть бути вирішені за допомогою рівняння методу граничних елементів згідно представленої методики з урахуванням деформацій вигину і розтягування-стиснення.
Ключові слова: кругові арки, метод граничних елементів, напружено-деформований стан, Matlab, автоматизація розрахунку

V.F. Orobey, O.F. Daschenko, O.M. Lymarenko. **Automation of arches calculation.** The procedure of automation of calculation of the strained-deformed state of circular arches is considered in the calculation of bending and tensile-compression deformations concentrated and distributed external loads. The aim of the work is to apply the possibilities of the boundary element method (BEM) to solve quite labor-intensive tasks of the strained-deformed state of circular arches and arch structures. To achieve the goal, a static calculation of the tensely-deformed state of circular arches in the MATLAB environment is performed. For this purpose, a system of differential equations of flat deformation of a circular rod is made and solved taking into account bending and stretching deformations along radial and tangential displacements. As a result of the calculation, it was concluded that numerous problems in the calculation of rings and ring systems can be solved by means of the boundary element method (BEM) equation in a coherently presented method, taking into account bending and tensile-compression deformations.
Keywords: circular arch, boundary element method (BEM), stress-strain state, MATLAB, automated calculation

Introduction. Curved rods (arches) are widely used in rocket engineering, aircraft and shipbuilding, bridges, mechanical engineering and construction. This is due to the advantages of curved rods in front of rectilinear rods due to their higher bearing capacity and rigidity.

However, the calculation of the stress-strain state of the arches is quite complicated, because it is necessary to take into account simultaneously deformation of bending, stretching-compression and shear. Especially it concerns the definition of movements in arches, because the Vereshchagin formula is not applicable here, since the freight and unit diagrams are nonlinear.

In this case we have to use the Morale integral, the Castigliano theorem, and numerical methods. To simplify the calculation of arches, we propose to use the numerical-analytical variant of the method of boundary elements [1] and the MATLAB programming environment.

In the literature, solutions are given for various problems of plane deformation of a circular rod with only bending deformation [2]. In 1938, Professor N.K. Snitko obtained a solution to the problem of plane deformation of a circular rod with allowance for bending and tension-compression deformations only for a particular load case: $q_y(\alpha)=q=$const, where q is the intensity of the vertical load.

The lack of a sufficiently accurate analytical solution of the problem of plane deformation of a circular rod has contributed to the fact that in a number of works [3 – 5] it is recommended to replace curvilinear rods by a set of rectilinear rods.

This model gives an error of not more than 1.0 % if a rectilinear rod draws an arc of a curved rod less than 5° [6]. This means that the ring can be represented by a regular polygon of 72 rods, an arch in 180° – 36 rods, etc. Further, the calculation of the curvilinear rod can be performed by the method of boundary elements (BEM), the force method, and other methods.

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If you apply the boundary element method and the MATLAB environment, you can significantly reduce the complexity of the calculation, simplify the algorithm and calculation schemes, increase the accuracy and reliability of the results, and introduce this methodology into the calculation practice and educational process of higher education institutions without much difficulty.

In connection with this, the goal of the paper is the practical application of the possibilities of the method of boundary elements for solving rather laborious problems on the stress-strain state of circular arches and arch structures.

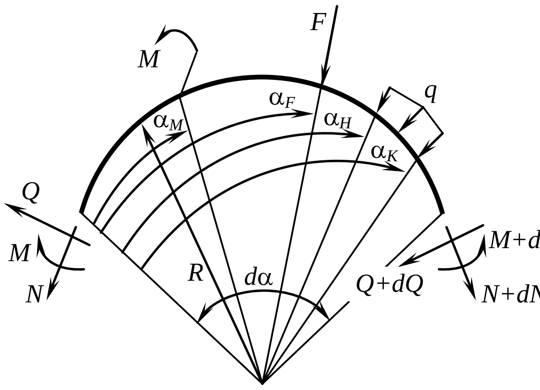

Fig. 1. Calculation scheme of the arch

Materials and methods. To achieve this goal, we will calculate the stress-strain state of circular arches in the MATLAB environment.

We show in Fig. 1 the calculation scheme of the arch and write down the basic equations needed for the calculation.

The system of differential equations for plane deformation of a circular rod with allowance for deformation of bending and extension with respect to radial $\vartheta(\alpha)$ and tangential displacement $u(\alpha)$ has the form:

$$\begin{cases} \vartheta^{\mathrm{IV}}(\alpha) + \dfrac{EAR^2}{EI}\vartheta(\alpha) + u'''(\alpha) - \dfrac{EAR^2}{EI}u'(\alpha) = \dfrac{R^4}{EI}q_y(\alpha), \\ \left(1 + \dfrac{EAR^2}{EI}\right)u''(\alpha) + \vartheta'''(\alpha) - \dfrac{EAR^2}{EI}\vartheta'(\alpha) = -\dfrac{R^4}{EI}q_x(\alpha), \end{cases} \tag{1}$$

where EA – Rigidity of section of arch at tension-compression, kN;

EI – Rigidity of section of the arch at bending, kNm^2;

R – Radius of the axis of the arch, m;

α – Angular coordinate, Rad.;

$q_y(\alpha)$ – Radial loading of the arch (along the normal to the axis), kN/m;

$q_x(\alpha)$ – Tangential loading of the arch, kN/m.

The solution of the Cauchy exercise on plane arch deformation can be represented in the matrix form as follows [1]:

			1	2	3	4	5	6				
$EIv(\alpha)$		1	A_{11}	A_{12}	$-A_{13}$	$-A_{14}$	A_{15}	A_{16}	$EIv(0)$		B_{11}	
$EI\varphi(\alpha)$		2		A_{22}	$-A_{23}$	$-A_{13}$		A_{26}	$EI\varphi(0)$		B_{21}	
$M(\alpha)$	=	3			A_{22}	A_{12}		$-A_{36}$	$M(0)$	+	$-B_{31}$	(2)
$Q(\alpha)$		4				A_{11}		$-A_{46}$	$Q(0)$		$-B_{41}$	
$EAu(\alpha)$		5	A_{51}	A_{52}	$-A_{53}$	$-A_{54}$	A_{11}	A_{56}	$EAu(0)$		$-B_{51}$	
$N(\alpha)$		6				$-A_{64}$		A_{11}	$N(0)$		$-B_{61}$	

where $EI\varphi(\alpha)$ – Angle of rotation of the section of the arch, kNm^2;

$M(\alpha)$ – Bending moment in section, kNm;

$Q(\alpha)$ – Transverse force in section, kN;

$N(\alpha)$ – Normal force in section, kN.

Fundamental orthonormal functions and terms from the radial load (Fig. 1) after integration take the form [1]:

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$$A_{11}=\cos\alpha;\ A_{12}=R\sin\alpha;\ A_{13}=R^2(1-\cos\alpha);\ A_{14}=\left(R^3+\frac{EIR}{EA}\right)\frac{\sin\alpha-\alpha\cdot\cos\alpha}{2};\ A_{15}=-\frac{EI}{EA}\sin\alpha;$$

$$A_{16}=R^3\left(1-\frac{1}{2}\alpha\cdot\sin\alpha-\cos\alpha\right)-\frac{EIR}{EA}\frac{1}{2}\alpha\cdot\sin\alpha;\ A_{22}=1;\ A_{23}=R\alpha;\ A_{26}=R^2(\alpha-\sin\alpha);$$

$$A_{36}=R(1-\cos\alpha);\ A_{46}=\sin\alpha;\ A_{51}=\frac{EA}{EI}\sin\alpha;\ A_{52}=\frac{EAR}{EI}(1-\cos\alpha);\ A_{53}=\frac{EAR^2}{EI}(\alpha-\sin\alpha);$$

$$A_{54}=\frac{EAR^3}{EI}\left(1-\cos\alpha-\frac{1}{2}\alpha\cdot\sin\alpha\right)-R\frac{1}{2}\alpha\cdot\sin\alpha;$$

$$A_{56}=R\frac{1}{2}(\sin\alpha+\alpha\cdot\cos\alpha)+\frac{EAR^2}{EI}\left(\alpha+\frac{1}{2}\alpha\cdot\cos\alpha-\frac{3}{2}\sin\alpha\right);\ A_{64}=-\sin\alpha;$$

$$\begin{aligned}
B_{11}=\left(R^4+\frac{EIR^2}{EA}\right)\Bigg\{&\frac{M}{R^2}\frac{(\alpha-\alpha_M)_+\cdot\sin(\alpha-\alpha_M)_+}{2}+\frac{F_y}{R}\frac{\sin(\alpha-\alpha_F)_+-(\alpha-\alpha_F)_+}{2}+\\
&+q_y\Big[H(\alpha-\alpha_H)-\cos(\alpha-\alpha_H)_+-\frac{1}{2}(\alpha-\alpha_H)_+\cdot\sin(\alpha-\alpha_H)_+\\
&-H(\alpha-\alpha_K)+\cos(\alpha-\alpha_K)_+\cdot\sin(\alpha-\alpha_K)_+\Big]\Bigg\};\\
B_{21}=MR&\sin(\alpha-\alpha_M)_++F_yR^2[H(\alpha-\alpha_F)-\cos(\alpha-\alpha_F)_+]+\\
&+q_yR^3[(\alpha-\alpha_H)_+-\sin(\alpha-\alpha_H)_+-(\alpha-\alpha_H)_++\sin(\alpha-\alpha_F)_+]\\
B_{31}=M&\cos(\alpha-\alpha_M)_++F_yR\sin(\alpha-\alpha_F)_++q_yR^2[H(\alpha-\alpha_H)-\cos(\alpha-\alpha_H)_+-\\
&-H(\alpha-\alpha_K)+\cos(\alpha-\alpha_K)_+];\\
B_{41}=\frac{M}{R}&[-\sin(\alpha-\alpha_M)_+]+F_y\cos(\alpha-\alpha_H)_++\\
&+q_yR[\sin(\alpha-\alpha_H)_+-\sin(\alpha-\alpha_K)_+];\\
B_{51}=M&\left[\left(1+\frac{EIR^2}{EI}\right)\frac{\sin(V-\alpha_m)_++(\alpha-\alpha_m)_+\cdot\cos(\alpha-\alpha_m)_+}{2}-\frac{EAR^2}{EI}\cdot\sin(\alpha-\alpha_M)_+\right]+F_y\times\\
&\times\left\{\left(1+\frac{EAR^2}{EI}\right)R^2\frac{\sin(\alpha-\alpha_H)_+-(\alpha-\alpha_H)_+}{2}-\frac{EAR^3}{EI}[H(\alpha-\alpha_F)-\cos(\alpha-\alpha_F)]\right\}+q_y\times\\
&\times\left\{\left(1+\frac{EAR^2}{EI}\right)R^2\frac{\sin(\alpha-\alpha_H)_+-(\alpha-\alpha_H)_+\cos(\alpha-\alpha_H)-\sin(\alpha-\alpha_K)_++(\alpha-\alpha_K)_+\cos(\alpha-\alpha_K)_+}{2}-\right.\\
&\left.-\frac{EAR^4}{EI}[(\alpha-\alpha_H)_+-\sin(\alpha-\alpha_H)_+-(\alpha-\alpha_K)_++\sin(\alpha-\alpha_K)_+]\right\};\\
B_{61}=\frac{M}{R}&\cos(\alpha-\alpha_M)_++F_y\sin(\alpha-\alpha_F)_++\\
&+q_yR[H(\alpha-\alpha_H)_+-\cos(\alpha-\alpha_H)_+-H(\alpha-\alpha_K)+\cos(\alpha-\alpha_K)_+].
\end{aligned}\qquad(3)$$

The next step is to the account of the boundary conditions for supporting the arches.

To determine the parameters of the stressed-deformed state of the arch, it is necessary to compile and solve a boundary value task that takes into account the given support conditions for boundary points. On the BEM, this equation has the form [1]

$$A_* X_* = -B, \tag{4}$$

where A_* – Matrix of boundary values of fundamental functions, which takes into account the boundary conditions of the arch;

X_* – Matrix of initial and final parameters of arch bending and stretching;

B – Load matrix with boundary variable value α.

Consider the various conditions of arch support.

With the hinged support, the diagram of which is shown in Fig. 2, constituted the matrices of the initial (X) and final (Y) parameters of the arch (5), where we took into account the hinged support of two boundary sections.

	X_*		Y
1	$EI\vartheta(0)=0$; $EI\varphi(\alpha_r)$	1	$EI\vartheta(\alpha_r)=0$
2	$EI\varphi(0)$	2	$EI\varphi(\alpha_r)$
3	$M(0)=0$; $Q(\alpha_r)$	3	$M(\alpha_r)=0$
4	$Q(0)$	4	$Q(\alpha_r)$
5	$EIu(0)=0$; $N(\alpha_r)$	5	$EIu(\alpha_r)=0$
6	$N(0)$	6	$N(\alpha_r)$

(5)

From these matrices it follows that in the matrix A_*, the columns 1, 3 and 5 must be zeroed and add the elements $A(2.1) = -1$; $A(4.3) = -1$; $A(6.5) = -1$, taking into account the transfer of parameters from Y to X_*. The matrix equation of the boundary value task for the arch in Fig. 2 becomes:

	(1)	2	(3)	4	(5)	6			
1		A_{12}		$-A_{14}$		A_{16}	$EI\varphi(\pi)$		$-B_{11}(\pi)$
2	-1	A_{22}		$-A_{13}$		A_{26}	$EI\varphi(0)$		$-B_{21}(\pi)$
3				$-A_{12}$		$-A_{36}$	$Q(\pi)$	=	$B_{31}(\pi)$
4			-1	$-A_{11}$		$-A_{46}$	$Q(0)$		$B_{41}(\pi)$
5		A_{52}		A_{54}		$-A_{56}$	$N(\pi)$		$B_{51}(\pi)$
6				A_{64}	-1	$-A_{64}$	$N(0)$		$B_{61}(\pi)$

(6)

After solving equation (6), the reactions of the arch will be: $H_0 = Q_{(0)}$; $R_0 = N_{(0)}$; $H_K = Q_{(\pi)}$; $R_K = N_{(\pi)}$, and the stress-strain state at internal points is determined by equation (2).

Similarly, making matrices X_*, Y, we get the equation of the boundary value task for the other case – hard pinching and hinged support, shown in Fig. 3.

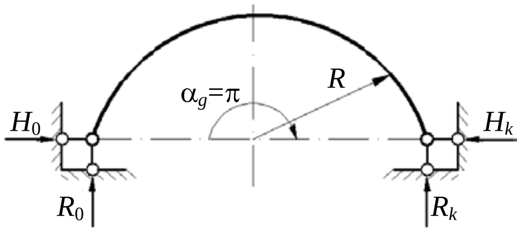

Fig. 2. Hinged arch

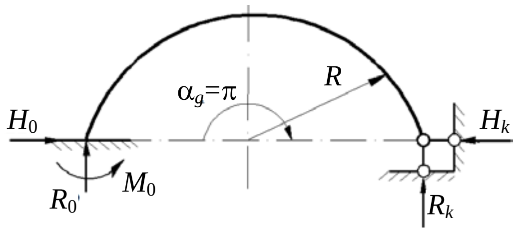

Fig. 3. Hard pinching and hinged support of arches

	(1)	(2)	3	4	(5)	6			
1			$-A_{13}$	$-A_{14}$		A_{16}	$EI\varphi(\pi)$		$-B_{11}(\pi)$
2	-1		$-A_{23}$	$-A_{13}$		A_{26}	$Q(\pi)$		$-B_{21}(\pi)$
3			A_{22}	A_{12}		$-A_{36}$	$M(0)$	=	$B_{31}(\pi)$
4		-1		A_{11}		$-A_{46}$	$Q(0)$		$B_{41}(\pi)$
5			$-A_{53}$	$-A_{54}$		A_{56}	$N(\pi)$		$B_{51}(\pi)$
6				$-A_{64}$	-1	A_{11}	$N(0)$		$B_{61}(\pi)$

(7)

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After solving (7), the reactions of the arch will be equal to

$$H_0 = Q_{(0)};\ R_0 = N_{(0)};\ H_K = Q_{(\pi)};\ R_K = N_{(\pi)};\ M_0 = M_{(0)}.$$

Fig. 4 shows a diagram of the following type of support for arches – the hard pinching of two boundary points.

The matrix equation of the boundary value task of this case of support will take the form (8).

	(1)	(2)	3	4	(5)	6				
1			$-A_{13}$	$-A_{14}$		A_{16}	$M(\pi)$		$-B_{11}(\pi)$	
2	-1		$-A_{23}$	$-A_{13}$		A_{26}	$Q(\pi)$		$-B_{21}(\pi)$	
3			A_{22}	A_{12}		$-A_{36}$	$M(0)$	=	$B_{31}(\pi)$	(8)
4		-1		A_{11}		$-A_{46}$	$Q(0)$		$B_{41}(\pi)$	
5			A_{53}	$-A_{54}$		A_{56}	$N(\pi)$		$B_{51}(\pi)$	
6				$-A_{64}$	-1	A_{11}	$N(0)$		$B_{61}(\pi)$	

Reactions of the arch will be equal

$$H_0 = Q_{(0)};\ R_0 = N_{(0)};\ M_0 = M_{(0)};\ H_K = Q_{(\pi)};\ R_K = N_{(\pi)};\ M_k = M_{(\pi)}.$$

Let us consider an example of determining the parameters of the stress-strain state of an arch with an arbitrary load and fixing.

Results. Define the stress-strain state of the arch (Fig. 5). The results are presented numerically (in the form of a table) and visually (in the form of diagrams).

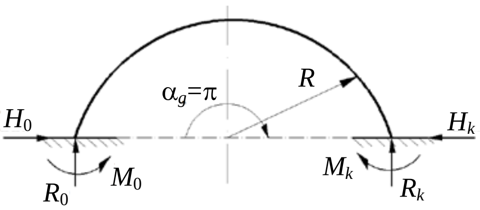

Fig. 4. Hard pinching of the arch

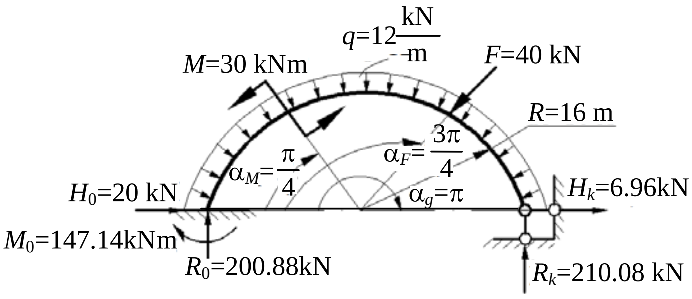

Fig. 5. Load and fixing in the arch

Angular coordinates for the concentrated moment and the concentrated force are $\alpha_M = \frac{\pi}{4};\ \alpha_F = \frac{3}{4}\pi$. The angular coordinates of the uniformly distributed load are $\alpha_H = 0;\ \alpha_K = \pi$.

With the given initial data, we calculate the values of the boundary parameters using Eq. (7) and represent the obtained results in Table 1.

Table 1

The value of the boundary parameters of the arch in Fig. 5

№ n/n	Parametr	Value
1	$EI\varphi(\pi)$, kNm^2	–958.34
2	$Q(\pi)$, kN	–6.96
3	$M(0)$, kNm	147.14
4	$Q(0)$, kN	–20.00
5	$N(\pi)$, kN	–210.08
6	$N(0)$, kN	–200.88

The reactions of the arch supports will be equal to H_0=20.00 kN, R_0=200.88 kN, M_0=147.14 kNm, H_k=6.96 kN, R_k=210.08 kN.

The values of the parameters of the stress-strain state are summarized in Table 2.

Table 2

The results of arch calculating in numerical form

Angular coordinate		Parameters of the stress-strain state					
rad	grad	$Eav \cdot 10^{-3}$, kNm^2	$Ea\varphi \cdot 10^{-2}$, kNm^2	*M*, kNm	*Q*, kN	$EAu \cdot 10^{-4}$, kNm	*N*, kN
0.0	0.0	0.0	0.0	147.14	–20.0	0.0	–200.88
0.09	5	0.133	1.86	119.79	–19.15	–0.033	–202.59
0.17	10	0.497	3.35	93.72	–18.16	–0.092	–204.22
0.26	15	1.038	4.48	6915	–17.02	–0.200	–205.76
0.35	20	1.703	5.28	46.24	–15.76	–0.371	–207.19
0.44	25	2.445	5.78	25.19	–14.38	–0.617	–208.50
0.52	30	3.214	6.00	6.15	–12.88	–0.943	–209.69
0.61	35	3.970	5.96	–10.74	–11.29	–1.348	–210.75
0.70	40	4.673	5.71	–25.34	–9.61	–1.831	–211.66
0.79	45	5.289	5.27	–67.55	–7.86	–2.383	–214.30
0.87	50	5.760	4.25	–77.16	–5.89	–2.993	–214.90
0.96	55	6.036	3.12	–83.98	–3.87	–3.643	–215.33
1.05	60	6.100	1.92	–87.96	–1.83	–4.310	–215.57
1.13	65	5.945	0.68	–89.07	0.24	–4.973	–215.64
1.22	70	5.570	–0.54	–87.30	2.30	–5.608	–215.53
1.31	75	4.980	–1.73	–82.67	04.34	–6.192	–214.24
1.40	80	4.189	–2.84	–75.20	6.35	–6.704	–214.78
1.48	85	3.219	–3.82	–64.9	8.31	–7.124	–214.14
1.57	90	2.095	–4.64	–52.03	10.21	–7.433	–213.33
1.66	95	0.853	–5.26	–36.50	12.03	–7.618	–212.36
1.75	100	–0.468	–5.65	–18.48	13.76	–7.668	–211.23
1.83	105	–1.824	–5.77	1.87	15.38	–7.578	–209.96
1.92	110	–3.165	–5.59	24.41	16.89	–7.345	–208.55
2.01	115	–4.435	–5.08	48.96	18.26	–6.975	–207.02
2.09	120	–5.578	–4.21	–75.35	19.50	–6.478	–205.37
2.18	125	–6.534	–2.96	103.36	20.59	–5.871	–203.62
2.27	130	–7.240	–1.32	132.79	21.53	–5.175	–201.78
2.36	135	–7.635	0.74	163.40	22.30	–4.421	–199.86
2.44	140	–7.673	2.85	139.20	–16.95	–3.645	–201.38
2.53	145	–7.383	4.63	116.14	–16.07	–2.882	–202.82
2.62	150	–6.813	6.10	94.39	–15.06	–2.165	–204.18
2.71	155	–6.008	7.28	74.13	–1394	–1.520	–205.44
2.79	160	–2.016	8.18	55.50	–12.72	–0.970	–206.61
2.88	165	–3.879	8.83	38.66	–11.40	–0.532	–207.66
2.97	170	–2.638	9.27	23.72	–9.99	–0.219	–208.59
3.05	175	–1.334	9.51	10.80	–8.50	–0.040	–209.40
3.14	180	0.0	9.58	0.0	–6.96	0.0	–210.08

The data in Table 2 indicate that the obtained results are accurate. This conclusion is justified by the fact that the stress-strain state of the arch was calculated from the equations of the method of initial parameters (2), where the initial parameters were taken from Table 1.

If the solution is exact, then the support boundary conditions must be satisfied on the right pedestal, which is confirmed by the data in Table 2.

The diagrams of the parameters of the stress-strain state of the arch in Cartesian coordinates are shown in Fig. 6 – 11.

Diagrams in Figures 6 – 11 possible easy to depict on the contour of the arch.

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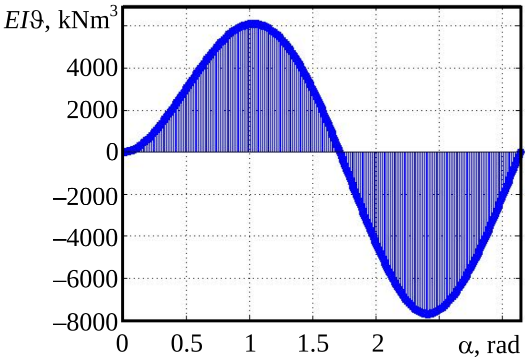

Fig. 6. Diagram of arch $EI\vartheta$, deflections, kNm^3

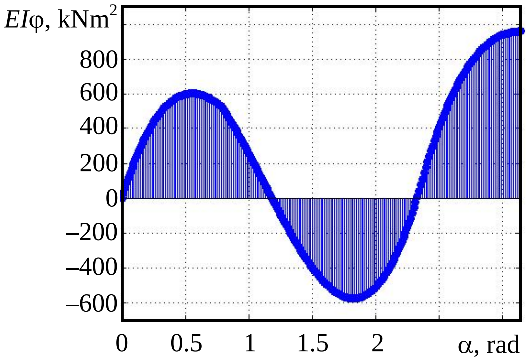

Fig. 7. Diagram of the rotation angles of the arch EIφ, kNm^2

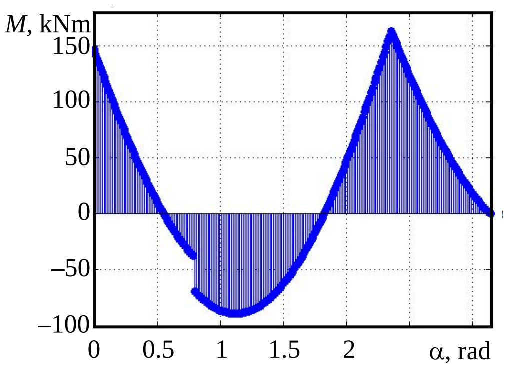

Fig. 8. Diagram of bending moments M, kNm

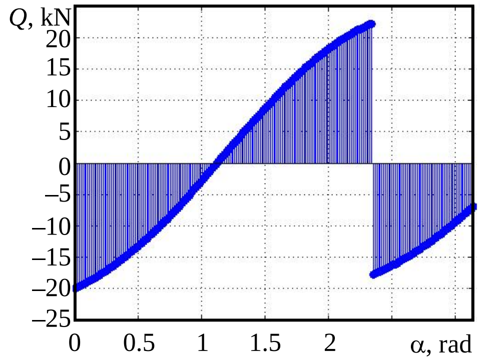

Fig. 9. Diagram of transverse forces Q, kN

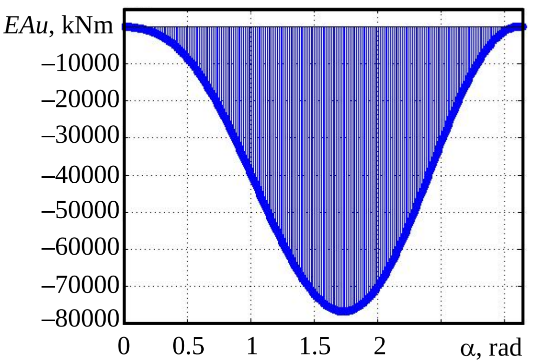

Fig. 10. Diagram of tangential displacements EAu, kNm

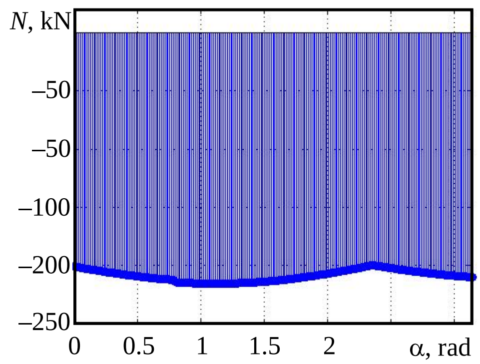

Fig. 11. Diagram of normal forces N, kN

Conclusions. Equations of boundary value tasks for determining unknown initial parameters of circular arcs under the existing conditions of support of boundary sections are obtained. On the basis of fundamental concepts of material resistance, dynamics and strength of structures a technique for calculating the stress-strain state of circular arches in the MATLAB environment is developed. Taking into account the effect of distributed and concentrated loads the calculation of a circular arch is performed.

It should be noted that the formation of the equations of boundary value tasks for the calculation of arches with different support conditions significantly shortens the time of other researchers in solving problems of calculating and analyzing structures containing circular arches.

The technique of calculating of the stress-strain state presented by the authors has significant advantages over the known methods. It describes in sufficient detail the calculation of the action of the external concentrated moment and the calculation of all the kinematic parameters of the deformed state of the circular arch.

In conclusion, we note that numerous tasks of calculating rings and ring systems [2, vol. 1, p. 321; 365] can be solved using BEM equation (2) in similar manner, but in a more exact formulation given bending and stretching-compression deformations.

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