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MODELLING AND SOLUTION OF CONTACT PROBLEM FOR INFINITE PLATE AND CROSS-SHAPED EMBEDMENT

О.Б. Козін, О.Б. Папковська, М.О. Козіна. **Моделювання і розв'язання контактної задачі для нескінченної пластини і хрестоподібного включення.** Розробка ефективних методів визначення напружено-деформованого стану тонкостінних конструкцій з включеннями, підкріпленнями й іншими концентраторами напружень є важливим завданням як з теоретичної, так і з практичної точки зору, враховуючи їх велике практичне застосування. ***Мета:*** Метою дослідження є розробка аналітичного математичного методу вивчення напружено-деформованого стану нескінченної пластини з хрестоподібним включенням при вигині. ***Матеріали і методи:*** Метод граничних елементів є ефективним способом розв'язання крайових задач для систем диференціальних рівнянь. Методи, засновані на граничних інтегральних рівняннях, знаходять широке застосування в багатьох галузях науки і техніки, включаючи розрахунок пластин і оболонок. Одним із методів розв'язання численного класу інтегральних рівнянь і систем, що виникають на базі методу граничних інтегральних рівнянь, є аналітичний метод зведення цих рівнянь і систем до задач Рімана з подальшим їх розв'язанням. ***Результати:*** Отримано інтегральне рівняння для аналізу прогинів і аналізу напружено-деформованого стану тонкої пружної пластини з жорстким хрестоподібним включенням. Зведенням до задачі Рімана і її подальшим розв'язанням отримано точний розв'язок даної крайової задачі. Досліджено асимптотичну поведінку контактних зусиль на кінцях включення.

Ключові слова: крайова задача, ізотропна пластина, жорстке хрестоподібне включення, вигин, перетворення Мелліна, метод факторизації, задача Рімана.

O.B. Kozin, O.B. Papkovskaya, M.O. Kozina. **Modelling and solution of contact problem for infinite plate and cross-shaped embedment.** Development of efficient methods of determination of an intense-strained state of thin-walled constructional designs with inclusions, reinforcements and other stress raisers is an important problem both with theoretical, and from the practical point of view, considering their wide practical application. ***Aim*:** The aim of this research is to develop the analytical mathematical method of studying of an intense-strained state of infinite plate with cross-shaped embedment at a bend. ***Materials and Methods*:** The method of boundary elements is an efficient way of the boundary value problems solution for systems of differential equations. The methods based on boundary integral equations get wide application in many branches of science and technique, calculation of plates and shells. One of methods of solution of a numerous class of the integral equations and systems arising on the basis of a method of boundary integral equations is the analytical method of construction of these equations and systems to Riemann problems with their forthcoming decision. ***Results*:** The integral equation for the analysis of deflections and the analysis of an intense-strained state of a thin rigid plate with rigid cross-shaped embedment is received. The precise solution of this boundary value problem is received by reduction to a Riemann problem and its forthcoming solution. An asymptotical behavior of contact efforts at the ends of embedment is investigated.

Keywords: boundary problem, isotropic plate, rigid cross-shaped embedment, bend, Mellin transform, factorization method, Riemann problem.

Introduction. Development of efficient methods of determination of an intense-strained state of thin-walled constructional designs with inclusions, reinforcements and other stress raisers is an important problem both with theoretical, and from the practical point of view, considering their wide practical application. Plates, reinforced by a different inclusions and ribs, are widely used in practice as components of different constructions. In this study we proposed a method of analytical solution for boundary value problem of stress-strain state of the bending of an infinite plate with a rigid cross-shaped embedment. The exact solution of this boundary value problem is obtained by reduction to the Riemann problem and by its subsequent solution.

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The boundary element method is an effective way of solving boundary value problems for systems of differential equations. Methods based on the boundary integral equations, are a powerful tool in many fields of science and technology, including the calculation of plates and shells [1…5].

However, due to the singularity of the fundamental solutions, a problem associated with irregular borders (corners, edges, etc...) arises. So, the question to use of special techniques for solving the problems with non-smooth boundary is actual.

One of methods for solving numerous classes of integral equations and systems, arising on the basis of the method of boundary integral equations, is an analytical method of reducing these equations and systems to the Riemann problem with their subsequent solution [6…13].

This method was further developed in solving the problem of bending isotropic [6] and orthotropic plates [7, 8] with linear irregularities oriented arbitrarily.

Contact problem of bandpass orthotropic plate Kirchhoff model with a thin semi-infinite rigid reinforcement were studied and solved in [9] by present method, as well as with reinforcement in the form of elastic rib [10].

In [11], an exact solution of the antisymmetric contact problem of bending bandpass orthotropic semi-infinite plate and a rigid support was constructed by reduction to the Riemann problem. The asymptotic behavior of the contact forces at the end of this support has been investigated.

Exact solution of the boundary value problem of bending bandpass shallow shell, which is supported by intermediate thin semi-infinite rib, type Winkler foundation was obtained in [12]; and supported by intermediate thin semi-infinite rigid support, was obtained in [13].

The aim of this research is to develop the analytical mathematical method of studying of an intense-strained state of infinite plate with cross-shaped embedment at a bend. It is also necessary to investigate the asymptotic behavior of the contact forces at the ends of this embedment.

Materials and Methods. We consider the problem of the bending of an infinite plate $(-\infty < x, y < \infty)$, containing a cross-shaped, thin, absolutely rigid embedment $(|x| \le a, y = 0; |y| \le a, x = 0)$.

The force P applied to the embedment in point $x = 0, y = 0$. P is an applied transverse load. The plate is simply supported in $4N$ points

$$M_k = (b\cos(\pi k/(2N) + \pi/(4N)), b\sin(\pi k/(2N) + \pi/(4N))) = (x_k, y_k),\ (b > a).$$

It is necessary to find the deflection of embedment W_0 and the contact forces of interactions $\psi_1(\xi), \psi_2(\xi)$ between embedment and plate.

Using the results of [6], we give mathematical formulation of the boundary problem described above. Equation, governing the deflection of mid-surface of plate $w(x, y)$ can be approximated as:

$$D\Delta^2 w(x,y) = \psi_1(x)\delta(y) + \psi_2(y)\delta(x) - \frac{P}{4N}\sum_{k=1}^{4N}\delta(x - x_k)\delta(y - y_k)\,; \tag{1}$$

The boundary conditions are the following:

$$\frac{\partial^3 w(x,y)}{\partial x^i \partial y^{3-i}} \to 0,\ i = \overline{0,3} \quad \text{as} \quad x^2 + y^2 \to \infty\,; \tag{2}$$

Moreover:

$$w(x,0) = W_0, \quad (|x| \le a); \qquad w(0,y) = W_0, \quad (|y| \le a;)\,; \tag{3}$$

$$\int_{-a}^{a}\psi_1(\xi)d\xi + \int_{-a}^{a}\psi_2(\xi)d\xi = P\,, \tag{4}$$

where $\delta(x),\ \delta(y)$ — Dirac delta functions.

Using the fundamental solution of the biharmonic equation $\Phi(x,y)=(8\pi)^{-1}(x^2+y^2)\ln(\sqrt{x^2+y^2})$, we obtain the solution of equation (1), satisfying (2).

$$Dw(x,y)=\int_{-a}^{a}\psi_1(\xi)\Phi(x-\xi,y)d\xi+\int_{-a}^{a}\psi_2(\eta)\Phi(x,y-\eta)d\eta-$$
$$-\frac{P}{4N}\sum_{k=1}^{4N}\Phi\left(x-b\cos\left(\frac{\pi k}{2N}+\frac{\pi}{4N}\right),\ y-b\sin\left(\frac{\pi k}{2N}+\frac{\pi}{4N}\right)\right). \tag{5}$$

Substituting (5) in (3), we obtain a system of two integral equations for $\psi_1(\xi)$ and $\psi_2(\eta)$. Posed problem is symmetrical relative to the variables x and y.

Therefore $\psi_1(\xi)=\psi_2(\xi)$. $\psi_1(\xi)$ — is even, and eventually we come to an integral equation of the first kind with a smooth kernel:

$$\frac{1}{8\pi D}\int_0^a\psi_1(\xi)L(x,\xi)d_\xi=W_0+f(x),\ 0\le x\le a; \tag{6}$$

where $L(x,\xi)=(x-\xi)^2\ln|x-\xi|+(x+\xi)^2\ln(x+\xi)+(x^2+\xi^2)\ln(x^2+\xi^2)$;

$$f(x)=\frac{P}{4N}\sum_{k=1}^{4N}\Phi\left(x-b\cos\left(\frac{\pi k}{2N}+\frac{\pi}{4N}\right),b\sin\left(\frac{\pi k}{2N}+\frac{\pi}{4N}\right)\right).$$

Performing the differentiation (6) three times with respect to x and introducing the notation $\tau=a^{-1}\xi,\ t=a^{-1}x,\ \psi_1(\xi)=\psi(\tau)$, we come to a singular equation

$$-\frac{1}{4\pi D}\int_0^1\psi(\tau)g\left(\frac{t}{\tau}\right)\frac{d\tau}{\tau}=f_3(t),\ 0\le t\le 1; \tag{7}$$

where $g(y)=\dfrac{1}{1-y}-\dfrac{1}{1+y}-\dfrac{6y}{1+y^2}+\dfrac{4y^3}{(1+y^2)^2}$, $f_3(t)=f'''(x)$

It is important to note that the solution of equation (7), when substituted into the left side of the equation (6), in general, can give a function that is different from $W_0+f(x)$ on an even polynomial of the second order $A+Bx^2$. The necessary and sufficient conditions for the equality to zero of this polynomial will be equalities

$$\frac{1}{2\pi D}\int_0^a\psi_1(\xi)\xi^2\ln|\xi|d\xi=W_0+f(0);\ \frac{1}{\pi D}\int_0^a\psi_1(\xi)(\ln|\xi|+1)d\xi=f''(0). \tag{8}$$

The first is obtained by substituting of $x=0$ into (6). The second — by double differentiation of (d^2/dx^2) (6) with respect to x and subsequent substituting $x=0$ into result. To satisfy (8), we should be seeking $\psi(\tau)$ in such class of functions in which the homogeneous equation, corresponding (7), has two linearly independent solutions. As will show below, it is necessary to search $\psi(\tau)$ in class of functions with non-integrable singularity at the point $\tau=1$, and the corresponding integrals are understood in regularized sense [14].

We extend the definition of right-hand side of equation (7) as $1\le t<\infty$, using the unknown function $f_+(t)$. Introducing the notation

$$f_-(t)=\begin{cases}f_3(t), & (0\le t<1);\\ 0, & (1\le t<\infty),\end{cases}\qquad \psi_-(\tau)=\begin{cases}-(4\pi D)^{-1}\psi(\tau), & (0\le\tau<1);\\ 0, & (1\le t<\infty),\end{cases}$$

let rewrite (7) in the form

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$$\int_0^\infty \psi_-(\tau) g\left(\frac{t}{\tau}\right)\frac{d\tau}{\tau} = f_-(t) + f_+(t)\,, \qquad (0 \le t < \infty)\,. \tag{9}$$

We applying Mellin transform to the (9)

$$\mathrm{M}\left[f_\pm(t), \psi_-(t)\right] = \int_0^\infty [f_\pm(t), \psi_-(t)] t^{p-1} dt = [F^\pm(p), \Psi^-(p)],\ p = \delta + i\tau,$$

$$[f_\pm(t), \psi_-(t)] = \frac{1}{2\pi i}\int_{\delta - i\infty}^{\delta + i\infty} [F^\pm(p), \Psi^-(p)] t^{-p} dp\,, (0 \le t < \infty)\,.$$

Then we calculate $\mathrm{M}[g] = G(p)$ using the formula 3.241 [15]. As a result, we have the Riemann problem

$$\Psi^-(p) G(p) = F^-(p) + F^+(p)\,,\ -\infty < \operatorname{Im} p < \infty\,, \tag{10}$$

where $G(p) = -\pi tg\frac{\pi p}{2}\left(1 - \frac{p-2}{\sin\frac{\pi p}{2}}\right)$, $(\max(\alpha, 0) < \operatorname{Re} p = \sigma < 1)$.

α is determined by the asymptotic behavior of function $\psi_-(\tau) = O(\tau^{-\alpha})$ as $\tau \to 0$.

Problem (10) is solved by the factorization method [16] with the use of representations

$$-\pi tg\frac{\pi p}{2} = \mathrm{T}^+(p)\mathrm{T}^-(p),\ \mathrm{T}^+(p) = -\pi\frac{\Gamma\left(\frac{1-p}{2}\right)}{\Gamma\left(1 - \frac{p}{2}\right)},\ \mathrm{T}^-(p) = \frac{\Gamma\left(\frac{1+p}{2}\right)}{\Gamma\left(\frac{p}{2}\right)};$$

$$\mathrm{K}(p) = 1 - \frac{p-2}{\sin\frac{\pi p}{2}} = \mathrm{K}^+(p)\mathrm{K}^-(p),\ \mathrm{K}^\pm(p) = \exp\left(\pm\frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty}\frac{\ln(\mathrm{K}(s))}{s - p} ds\right);$$

$$G(p) = G^+(p) G^-(p),\ G^\pm(p) = \mathrm{T}^\pm(p)\mathrm{K}^\pm(p);$$

$$\mathrm{H}(p) = \frac{F^-(p)}{G^+(p)} = \mathrm{H}^+(p) - \mathrm{H}^-(p),\ \mathrm{H}^\pm(p) = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty}\frac{\mathrm{H}(s) ds}{s - p}\ .$$

As a result, (10) is transformed into

$$\Psi^-(p) G^-(p) + H^-(p) = F^+(p) / G^+(p) + H^+(p) = Q(p). \tag{11}$$

To obtain two constants satisfying the conditions (8), it is necessary to have $Q(p) = c_0 + c_1 p$.

Results. Thus, exact solutions of equations (7) and (6) have the next form

$$\psi(\tau) = 2\pi D\int_{\sigma - i\infty}^{\sigma + i\infty} (c_0 + c_1 p - \mathrm{H}^-(p))\frac{\tau^{-p}}{G^-(p)} dp\,, \tag{12}$$

where $\psi_1(\xi) = \psi(\xi a^{-1}) = 2\pi D[c_0\varphi_0(\xi) + c_1\varphi_1(\xi) + \varphi_*(\xi)]$;

$$\varphi_0(\xi) = \frac{i}{\pi}\int_{\sigma - i\infty}^{\sigma + i\infty}\frac{\xi^{-p}}{G^-(p)} dp;$$

$$\varphi_1(\xi) = \frac{i}{\pi}\int_{\sigma - i\infty}^{\sigma + i\infty}\frac{p\xi^{-p}}{G^-(p)} dp;$$

$$\varphi_*(\xi)=\frac{-i}{\pi}\int_{\sigma-i\infty}^{\sigma+i\infty} H^-(p)\frac{\xi^{-p}}{G^-(p)}dp.$$

Here the constants c_0, c_1, W_0 are found from the equations (8) and (4):

$$c_0\int_0^a \varphi_0(\xi)\xi^2 \ln|\xi|d\xi + c_1\int_0^a \varphi_1(\xi)\xi^2 \ln|\xi|d\xi + \int_0^a \varphi_*(\xi)\xi^2 \ln|\xi|d\xi = W_0 + f(0);$$

$$c_0\int_0^a \varphi_0(\xi)(\ln|\xi|+1)d\xi + c_1\int_0^a \varphi_1(\xi)(\ln|\xi|+1)d\xi + \int_0^a \varphi_*(\xi)\ (\ln|\xi|+1)d\xi = \frac{f''(0)}{2};$$

$$c_0\int_0^a \varphi_0(\xi)d\xi + c_1\int_0^a \varphi_1(\xi)d\xi + \int_0^a \varphi_*(\xi)d\xi = \frac{\mathrm{P}}{8\pi D}.$$

Conclusions. Mathematical formulation of the boundary value problem is done. An integral equation for stress-strain analysis of thin supported elastic plate with rigid cross-shaped embedment is obtained. The exact solution of this boundary value problem is obtained by reduction to the Riemann problem and by its subsequent solution.

The behavior of the function $\psi_1(\xi)$ when $\xi \to a-0$ (defined by the asymptotic behavior $\Psi^-(p)$ at $|p|\to\infty$) is of great interest. Since $p[G(p)]^{-1} = \mathrm{O}(p^{1/2})$ then, according to [17] $\psi_1(\xi) = \mathrm{O}((a-\xi)^{-3/2})$, i.e., the contact forces have a non-integrable singularity at the ends of the cross-shaped embedment. This singularity coincides with the result in [9].

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