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# PECULIARITIES OF MATHEMATICAL MODELING OF AN INDUCTION MOTOR TAKING INTO ACCOUNT ITS NONLINEARITIES

*А. Бойко, О. Бесараб, В. Пліс.* **Особливості математичного моделювання асинхронного двигуна з урахуванням його нелінійностей.** Виконано вибір математичного опису асинхронного двигуна. Запропоновано моделювання асинхронного двигуна в трифазних природних осях А, В, С з урахуванням нелінійності параметрів асинхронного двигуна. Мета дослідження спрямована на підвищення відповідності моделі асинхронного двигуна реальному двигуну за рахунок урахування нелінійностей його параметрів. Математична модель, отримана з урахуванням припущень та ідеалізації, являє собою систему диференціальних, алгебраїчних та логічних рівнянь, що відображають умови електричної та механічної рівноваги та умови електромеханічного перетворення енергії. Рівняння електричної рівноваги складені за законами Кірхгофа та рівняннями Максвелла, а механічного – за рівняннями Даламбера та Лагранжа. В якості змінних стану використовуються потокозчеплення статора та ротора, що визначаються на кожному кроці чисельного інтегрування. Для адекватності моделі при розрахунках обов'язково необхідно врахувати нелінійності АД – ефекти витіснення струму і насичення машини, втрати у сталі, зміну робочої температури. Вибір нелінійності параметра, що враховується, а також методики урахування індивідуальні і визначаються складністю завдань, які ставляться перед моделлю. Повнота урахування нелінійностей параметрів АД визначається вимогами до точності дослідження та обов'язково передбачає урахування найбільш впливових на робочі характеристики машини.
*Ключові слова*: асинхронний двигун, математична модель, насичення АД, витіснення струму, нелінійність параметрів АД

*A. Boiko, O. Besarab, V. Plis.* **Peculiarities of mathematical modeling of an induction motor taking into account its nonlinearities.** The choice of the mathematical description of the induction motor has been made. Modeling of an induction motor in three-phase natural axes A, B, C is proposed, taking into account the nonlinearity of the parameters of an induction motor. The purpose of the study is aimed at improving the correspondence of the model of an induction motor to a real motor by taking into account the nonlinearities of its parameters. The mathematical model, obtained taking into account assumptions and idealization, is a system of differential, algebraic and logical equations that reflect the conditions of electrical and mechanical equilibrium and the conditions of electromechanical energy conversion. The electrical equilibrium equations are compiled according to Kirchhoff's laws and Maxwell's equations, and the mechanical ones – according to the d'Alembert and Lagrange equations. As state variables, the stator and rotor flux links are used, which are determined at each step of numerical integration. For the adequacy of the model in the calculations, it is necessary to take into account a number of non-linearities of the IM – the effect of current displacement and machine saturation, losses in steel, changes in operating temperature. The choice of the parameter nonlinearity to be taken into account, as well as the accounting methods, are individual and are determined by the complexity of the tasks that are set for the model. The completeness of taking into account the non-linear parameters of the IM is determined by the requirements for the accuracy of the study and necessarily provides for taking into account the most influencing the performance of the machine.
*Keywords*: induction motor, mathematical model, IM saturation, current displacement, non-linearity of IM parameters

## Introduction

All known electrical machines, including induction ones, can be described on the basis of the principles of a generalized machine. A generalized machine is an idealized non-equal-pole machine with symmetrical concentrated three-phase windings on the stator and rotor.

The simplest mathematical description of an induction machine can be performed in two-phase orthogonal coordinates. Known orthogonal coordinates: in axes rotating at arbitrary speed – $u$, $v$, 0, with synchronous speed – $x$, $y$, 0, with rotor speed – $d$, $q$ ,0 and stationary relative to the stator – $u$, $v$, 0. Application of one or another model is determined by the purpose and tasks of the research. The description of any of them is based on replacing a real three-phase machine with an equivalent two-phase model [1].

Thanks to the use of known semiconductor converters, the voltage applied to the stator windings of the induction motor (IM) is non-sinusoidal. There are two fundamentally different approaches to modeling IM with non-sinusoidal power supply. The first is based on decomposing the real voltage form into harmonic components and presenting the motor as a multi-stator and multi-rotor machine, each set of stator and rotor of which is powered by a certain voltage harmonic and has parameters cor-

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responding to this harmonic. The resulting moment is defined as the sum of individual components, taking into account the phase and direction of rotation of individual harmonics. In the second approach, it is considered that IM has one set of windings to which a real non-sinusoidal voltage is applied. The second approach is most often used to solve the tasks of electric drive (ED) research.

IM modeling in the natural three-phase axes *A*, *B*, *C* of a stationary coordinate system is more difficult to perform, but has a number of useful qualities that are not characteristic of modeling in orthogonal coordinates. When describing the axes of three-phase coordinates, they are combined with the magnetic axes of the phase winding of a real induction machine. In axes *A*, *B*, *C*, it is possible to simulate the operation of IM when powered by an asymmetric voltage source, as well as a number of emergency modes of IM. The completeness of consideration of the nonlinearities of IM parameters is determined by the requirements for the accuracy of the study and necessarily involves taking into account the most influential on the working characteristics of the machine.

**Analysis of recent research and publications**

Much attention has been paid to IM modeling [1 – 12]. However, a complete description of model algorithms is practically not found in the technical literature, which complicates the creation of software developments. In addition, a significant problem is taking into account the nonlinearities of IM parameters and semiconductor converters, which is necessary to fulfill the conditions of adequacy of the model to real electrotechnical processes [2]. Researchers suggest taking into account the change in active and reactive resistances due to the effects of current displacement and machine saturation, losses in steel, and changes in operating temperature [3]. The choice of nonlinearity of the parameter that is taken into account, as well as the methods of accounting, are individual and determined by the complexity of the tasks that are presented to the model. Some sources claim that to solve most research problems, it is enough to take into account the effects of saturation of the machine's magnetic circuits and current displacement [4].

Thermal calculation allows you to determine and optimize a number of parameters related to heat generation and heat removal. In solving problems related to the distribution of heat in the engine, the introduction of its thermal model helps. With its help, you can see the effect of the operating mode on engine heating [5]. The presence and consideration in the model of additional components related to heat exchange in the engine makes the model more accurate and more suitable for calculations. It is also important that the thermal model take into account the mode of operation of the electric motor, because the deviation of the main mode of operation from the nominal one will inevitably leIM to its overheating.

To date, the literature describes both simple thermal models of asynchronous electric motors, which give only an approximate idea of the heating of the motor, and very complex ones, where it is necessary to know specific parameters that are often known only to manufacturing plants [6, 7]. The basis of the simplest one-mass thermal model is the heat balance equation [8]. When building this model, the following assumptions are made – the electric motor is represented as a homogeneous mass with evenly distributed heat sources, with infinite internal thermal conductivity, with heat output proportional to the difference in temperature between the motor and the environment.

The advantages of the single-mass model include its simplicity and reliability of estimating the thermal state of the engine during its operation with constant power (losses in the engine $\Delta P$ and heat flow in the engine are constant) and constant ambient temperature [9]. The advantage of multi-mass models is the accurate prediction of the temperature at any point of the electric motor and the ability to take into account the mutual influence of all machine elements on the thermal state [10].

Disadvantages of such models [11]:

– to calculate the state of the model, it is necessary to know the specific parameters of the electric motors and complete data on the materials used (masses of copper, steel, etc.), which are usually known only to the manufacturing plants;

– it is necessary to measure the temperature at various points of the engine with high accuracy using the installed temperature sensors;

– with a large number of components, it is very difficult to set the parameters of such a model experimentally or by calculation in real time;

– solving the system of heat balance equations for each element of the asynchronous motor scheme is a difficult task and takes quite a lot of time.

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**The purpose of the work**: to perform the synthesis of the universal mathematical model of IM taking into account the nonlinearities of its parameters with the possibility of operation from semiconductor converters.

**Description of the mathematical model of IM**

Both the model in real three-phase coordinates and the model reduced to two-phase coordinates will be considered. The naturalness of the description and universality are important positive properties of the model in the natural three-phase axes *A*, *B*, *C* of the fixed coordinate system, which determines its choice for solving tasks of real work. Given that a real asynchronous motor is a complex of three-dimensional electric and magnetic circuits containing various nonlinearities, it is impossible to accurately describe it using Maxwell's equations. When modeling, the following assumptions are most acceptable: symmetry of the stator windings; symmetry of the rotor windings; sinusoidal distribution of magnetic fields along a circle; uniformity of the air gap; mutual independence of saturation along the scattering paths and through the main magnetic flux.

The stator and rotor flux linkages determined at each step of the numerical integration are chosen as variable states, and the stator and rotor voltage balance equations IM have the form [1]:

$$\left.\begin{aligned} u_{1AB} &= r_1 i_{1A} - r_1 i_{1B} + \dot{\psi}_{1AB} \\ u_{1BC} &= r_1 i_{1B} - r_1 i_{1C} + \dot{\psi}_{1BC} \\ u_{1CA} &= -r_1 i_{1A} + r_1 i_{1C} + \dot{\psi}_{1CA} \end{aligned}\right\}, \qquad (1)$$

where $u_{1AB,}\, u_{1BC,}\, u_{1CA}$ – linear voltages on the IM stator; $r_1$ – active resistance of the stator; $i_{1A}$, $i_{1B}$, $i_{1C}$ – stator phase currents; $\psi_{1AB}, \psi_{1BC}, \psi_{1CA}$ – linear flux linkages of the stator.

$$\left.\begin{aligned} 0 &= r_2 i_{2A} - r_2 i_{2B} + \dot{\psi}_{2AB} + \tfrac{1}{\sqrt{3}}\omega_2(\psi_{2BC} - \psi_{2CA}) \\ 0 &= r_2 i_{2B} - r_2 i_{2C} + \dot{\psi}_{2BC} + \tfrac{1}{\sqrt{3}}\omega_2(\psi_{2CA} - \psi_{2AB}) \\ 0 &= -r_2 i_{2A} + r_2 i_{2C} + \dot{\psi}_{2CA} + \tfrac{1}{\sqrt{3}}\omega_2(\psi_{2AB} - \psi_{2BC}) \end{aligned}\right\}, \qquad (2)$$

where $r_2$ – active resistance of the rotor; $i_{2A}$, $i_{2B}$, $i_{2C}$ – rotor phase currents; $\psi_{2AB}, \psi_{2BC}, \psi_{2CA}$ – linear flow couplings of the rotor; $\omega_2 = \omega_r p$ – electrical angular speed of the rotor, rad/s; $p$ – the number of IM pole pairs.

In a three-phase system with an isolated neutral, the sum of phase and line voltages, currents and flux linkages is zero. Therefore, the original differential equations of electrical balance for the stator and rotor circuits have only two independent variables each and the system of equations itself has redundancy. The calculation of the stator and rotor currents based on the original flux linkage equations is difficult due to the high order of the algebraic equations. In addition, when using application programs for solving matrix equations, there are conditions under which the main determinant is zero. This leads to division by zero and the impossibility of directly solving the system. But, given that:

$$\left.\begin{aligned} \psi_{1A} + \psi_{1B} + \psi_{1C} &= 0 \\ \psi_{2A} + \psi_{2B} + \psi_{2C} &= 0 \end{aligned}\right\}, \qquad (3)$$

the equations of electrical balance of the stator and rotor can be written in a more compact and convenient form:

$$\left.\begin{aligned} u_{1AB} &= r_1 i_{1A} - r_1 i_{1B} + \dot{\psi}_{1AB} \\ u_{1BC} &= r_1 i_{1B} - r_1 i_{1C} + \dot{\psi}_{1BC} \\ 0 &= r_2 i_{2A} - r_2 i_{2B} + \dot{\psi}_{2AB} + \tfrac{1}{\sqrt{3}}\omega_2(\psi_{2BC} - \psi_{2CA}) \\ 0 &= r_2 i_{2B} - r_2 i_{2C} + \dot{\psi}_{2BC} + \tfrac{1}{\sqrt{3}}\omega_2(\psi_{2CA} - \psi_{2AB}) \end{aligned}\right\}. \qquad (4)$$

After calculating the two unknown flux linkages, the third linear flux linkage is determined based on the expressions:

$$\left.\begin{aligned}\psi_{1CA} &= -(\psi_{1AB} + \psi_{1BC})\\ \psi_{2CA} &= -(\psi_{2AB} + \psi_{2BC})\end{aligned}\right\}. \tag{5}$$

One of the numerical methods can perform the solution of the electrical balance equations separately for each phase of the asynchronous machine or in the form of a system of equations. For software implementation of a mathematical model, it is convenient to use the mathematical apparatus of matrices:

$$\boldsymbol{\psi} = \mathbf{L}\times\mathbf{i}, \tag{6}$$

where $\boldsymbol{\psi}$ – column matrix of stator and rotor flux linkages; $\mathbf{i}$ – column matrix of stator and rotor currents; $\boldsymbol{L}$ is a square matrix of inductive coupling coefficients.

The column matrix of flow couplings looks like this:

$$\boldsymbol{\psi}^T = \left\|{}_{1AB}, \psi_{1BC}, \psi_{2AB}, \psi_{2BC}\right\|, \tag{7}$$

and the column of currents matrix:

$$\mathbf{i}^T = \left\|i_{1A}, i_{1B}, i_{1C}, i_{2A}, i_{2B}, i_{2C}\right\|. \tag{8}$$

Matrix of inductive coupling coefficients:

$$\mathbf{L} = \begin{Vmatrix} 1.5L_1 & -1.5L_1 & 1.5M_0 & -1.5M_0 \\ 1.5L_1 & 3L_1 & 1.5M_0 & 3M_0 \\ 1.5M_0 & -1.5M_0 & 1.5L_2 & -1.5L_2 \\ 1.5M_0 & 3M_0 & 1.5L_2 & 3L_2 \end{Vmatrix}, \tag{9}$$

where $L_1$, $L_2$ are self-induction coefficients of the stator and rotor windings; $M_0$ is the coefficient of mutual induction of the stator and IM rotor windings.

The coefficients of self-induction are determined by the parameters of the T-shaped substitution scheme of the asynchronous machine. After solving the system of equations $\boldsymbol{\psi} = \mathbf{L}\times\mathbf{i}$ according to the found currents $i_{1A}$, $i_{1B}$, $i_{2A}$, $i_{2B}$, the stator and rotor currents of the IM third phase are found:

$$\left.\begin{aligned} i_{1C} &= -(i_{1A} + i_{1B})\\ i_{2C} &= -(i_{2A} + i_{2B})\end{aligned}\right\}. \tag{10}$$

In the analysis of asymmetric modes, the expression of the electromagnetic moment is used, which takes into account the sum of the pairwise products of all IM currents [2]:

$$M_{\text{IM}} = \frac{x_0 p[(i_{1A}i_{2C} + i_{1B}i_{2A} + i_{1C}i_{2B}) - (i_{1A}i_{2B} + i_{1B}i_{2C} + i_{1C}i_{2A})]}{\sqrt{3}\cdot\omega_0}. \tag{11}$$

The IM model based on a two-phase generalized machine is based on the fact that a generalized three-phase vector of any variable value can be decomposed into the projection of any coordinate system rotating at an arbitrary speed. The simplest is the case with orthogonal coordinates.

A model of a generalized machine in vector form [1]:

$$\left.\begin{aligned} &\overline{U}_1 = R_1\cdot\overline{i}_1 + \dot{\overline{\Psi}}_1\\ &0 = R_2\cdot\overline{i}_2 + \dot{\overline{\Psi}}_2 - j\omega_r p\overline{\Psi}_2\\ &\overline{\Psi}_1 = L_1\cdot\overline{i}_1 + M_{12}\cdot\overline{i}_2\\ &\overline{\Psi}_2 = M_{12}\cdot\overline{i}_1 + L_2\cdot\overline{i}_2\\ &\dot{\omega}_r = \left(\left(1.5\,\mathrm{Im}\left(\overset{*}{\overline{\Psi}}_1\cdot i_1\right)\right) \pm M_C\right)\Big/ J_\Sigma \end{aligned}\right\}, \tag{12}$$

where $\overline{U}_1$, $\overline{i}_1$, $\overline{\Psi}_1$ are the generalized vectors of voltage, current and flux linkage of the stator; $\overset{*}{\overline{\Psi}}_1$ –the complex conjugate vector of the stator flux coupling; $\overline{i}_2$, $\overline{\Psi}_2$ – generalized vectors of current and flux coupling of the rotor; $j$ is the 90° rotation operator in the orthogonal coordinate systems α-β, *d-q* and *x-y*.

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From the model of the generalized machine when projecting the generalized vector on the axis of the coordinates fixed relative to the stator [1], the IM model in the system of coordinates fixed relative to the stator α-β is obtained:

$$\dot{\Psi}_{1\alpha} = U_{1\alpha} - i_{1\alpha} \cdot R_1;$$
$$\dot{\Psi}_{1\beta} = U_{1\beta} - i_{1\beta} \cdot R_1;$$
$$\dot{\Psi}_{2\alpha} = -i_{2\alpha} \cdot R_2 - \omega_r p \Psi_{2\beta};$$
$$\dot{\Psi}_{2\beta} = -i_{2\beta} \cdot R_2 + \omega_r p \Psi_{2\alpha}; \quad (13)$$
$$\begin{vmatrix} i_{1\alpha} \\ i_{1\beta} \\ i_{2\alpha} \\ i_{2\beta} \end{vmatrix} \times \begin{vmatrix} L_1 & 0 & M_{12} & 0 \\ 0 & L_1 & 0 & M_{12} \\ M_{12} & 0 & L_2 & 0 \\ 0 & M_{12} & 0 & L_2 \end{vmatrix} = \begin{vmatrix} \Psi_{1\alpha} \\ \Psi_{1\beta} \\ \Psi_{2\alpha} \\ \Psi_{2\beta} \end{vmatrix};$$
$$\dot{\omega}_r = (1.5 p M_{12}(i_{2\alpha} \cdot i_{1\beta} - i_{1\alpha} \cdot i_{2\beta}) \pm M_C) / J_\Sigma;$$
$$\dot{\alpha}_0 = \omega_0,$$

where α, β are the indices of the projections of the variables on the axis of the same name; $U_{1\alpha}$, $U_{1\beta}$ – voltage projections on the motor stator: $U_{1\alpha} = |\vec{U}_1| \cos(\alpha_0)$, $U_{1\beta} = |\vec{U}_1| \sin(\alpha_0)$; $\alpha_0$ is the rotation angle of the flow coupling vector; $\Psi_{1\alpha}$, $\Psi_{1\beta}$ – stator flux coupling; $\Psi_{2\alpha}$, $\Psi_{2\beta}$ – flux coupling of the rotor; $i_{1\alpha}, i_{1\beta}, i_{2\alpha}, i_{2\beta}$ – stator and rotor currents.

The disadvantage of this model is that it is designed to supply IM from a symmetrical sinusoidal voltage system. When feeding with asymmetric voltages, it is necessary to decompose the voltages into forward, reverse, zero sequence voltages, and consider the moment on the IM shaft as the sum of moments from each of these sequences. In addition, with a non-sinusoidal supply voltage for models in the coordinate systems associated with the stator or rotor fields, *d-q*, *x-y*, it is necessary to decompose the supply voltage into harmonic components. In addition, the moment on the IM shaft is considered as the sum of moments from each harmonic of the supply voltage, which naturally makes the calculation many times more difficult.

**Methods of taking nonlinearities of asynchronous motor parameters into account**

When designing asynchronous machines to reduce the mass of IM, the operating point of the nominal mode is selected on the non-linear section of the magnetization characteristic. That is, the machine is saturated. The IM model should take into account the change in the inductive resistance of the magnetization circuit in the entire range of changes in the magnetization current. Saturation can be taken into account using curves that determine the dependence of the resistance of the magnetization circuit on the magnetization current $X_\mu = f(I_\mu)$. To determine these dependencies, it is necessary to use experimental data. When modeling, it is convenient to apply the dependence of the change in the IM magnetization circuit parameters on the magnetization current, presented in relative units. An example of the results of the piecewise linear approximation of the dependence $X_\mu^* = f(I_\mu^*)$ for IM of AMU 4...11 kW series is presented in the Table 1.

**Table 1**

Results of piecewise linear approximation of the dependence $X_\mu^* = f(I_\mu^*)$

| Range of change $I_\mu^*$ | $X_\mu^*$ |
|---|---|
| $0 \le I_\mu^* \le 0.5$ | *1.35* |
| $0.5 \le I_\mu^* \le 1.0$ | $X_\mu^* = 1.35 - 0.5(I_\mu^* - 0.5)$ |
| $1.0 \le I_\mu^* \le 2.0$ | $X_\mu^* = 1.1 - 0.34(I_\mu^* - 1)$ |
| $2.0 \le I_\mu^* \le 4.5$ | $X_\mu^* = 0.76 - 0.16(I_\mu^* - 2.0)$ |
| $4.5 \le I_\mu^* \le 9.0$ | $X_\mu^* = 0.36 - 0.04(I_\mu^* - 4.5)$ |
| $9.0 \le I_\mu^* \le 17.0$ | $X_\mu^* = 0.2 - 0.01(I_\mu^* - 9.0)$ |
| $17 \le I_\mu^*$ | $X_\mu^* = 0.1$ |

In order to take into account the saturation by the IM main magnetic flux, it is necessary to determine the new value of the resistance of the magnetization circuit using the current correction factor $X_\mu^*$:

$$X_{\mu N} = X_\mu^* \cdot X_\mu, \tag{14}$$

where $X_\mu$ is the inductive resistance of the magnetization circuit without taking into account the saturation effect.

The multiplicity of the magnetizing current is determined by the ratio of the current value of the magnetizing current amplitude to the nominal value:

$$I_\mu^* = \frac{I_{\mu\,\max}}{I_{\mu\,\mathrm{nom}}}. \tag{15}$$

Nominal value of the magnetizing current:

$$I_{\mu\,\mathrm{nom}} = \frac{U_{\mathrm{nom}}}{\sqrt{(X_\mu + X_1)^2 + R_1^2}}, \tag{16}$$

where $U_{\mathrm{nom}}$ is the IM nominal voltage, $V$; $X_\mu$, $X_1$, $R_1$ are the parameters of the IM, Ohm substitution scheme.

The current value of the magnetizing current amplitude is determined at each step of the calculation:

$$I_{\mu\,\max} = \left(\frac{2 \cdot I_{\mu A}}{3} - \frac{I_{\mu B} + I_{\mu C}}{3}\right)^2 + \left(\frac{I_{\mu B} - I_{\mu C}}{\sqrt{3}}\right)^2, \tag{17}$$

where $I_{\mu A}$, $I_{\mu B}$ and $I_{\mu C}$ are the calculated magnetization currents of each phase of IM.

To take into account the saturation by scattering, it is possible to use the saturation coefficient $K_{\mathrm{sat}}=X_{1\mathrm{sat}}/X_{1\mathrm{nom}}$, which, in turn, depends on the value of the short-circuit current and can be specified in graphical or tabular form.

One of the simplified, but quite effective methods of taking into account the effect of current displacement in IM modeling is based on the use of rotor resistances in the short-circuit mode – $R_k$, $X_k$ [2]. In the future, these parameters are refined depending on the current slip value. During operation of IM in the field of nominal sliding, calculated resistances $R_n$, $X_n$ corresponding to the nominal mode of operation are used (index of resistance – $n$). The functional dependence of resistances from sliding in the region between $R_k$ and $R_n$, $X_k$ and $X_n$ is assumed to be linear.

During operation, the engine heats up due to the conversion of energy losses into heat. During its operation, the electric motor can heat up only to a certain, permissible temperature, which is determined primarily by the heat resistance of the insulating materials used. Compliance with the established limits on the permissible heating temperature ensures its standard service life of at least 20 years. Exceeding the permissible temperatures leads to the destruction of the insulation of the windings and shortens the life of the motor.

For motors, it is not the permissible temperature of the winding and other parts of the machine that is regulated, but the permissible excess of the temperature of the winding above the ambient temperature. Heating conditions of individual machine elements are different. Parts of the windings located in the inner regions of the machine are subject to greater heating. The release of heat in different operating modes is also uneven, and therefore the direction of heat flows inside the machine is not constant. At idle, heat is transferred from the more heated steel of the engine to its windings, and in the loaded state, the windings are more heated than the steel, and the direction of the heat flow is reversed [12]. Because of the heterogeneity of the engine as a physical body, the exact consideration of thermal processes in it turns out to be very difficult.

Thermal models have different degrees of detail. The simplest is the one-mass model, in which, when analyzing the heating and cooling processes, the engine is usually taken to be a continuous homogeneous body with an infinitely large thermal conductivity. In addition, it is believed that the amount of heat released into the environment is proportional to the temperature difference between the engine and the cooling medium; the cooling medium has an infinitely large heat capacity, that is, its

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temperature does not change during engine heating; heat loss, engine heat capacity and heat transfer coefficient are independent of engine temperature. In such a model, the heterogeneity of temperatures between the housing and its individual parts is not taken into account, but the reference point for temperature measurement is the part of the motor that is most prone to overheating, such is any of the stator windings.

The equation of the thermal balance of the engine at constant loIM has the form [13]:

$$Qdt = A\tau\, dt + C\, d\tau\,, \tag{18}$$

where $Q$ is the amount of heat (power loss in the engine) released by the engine per unit of time, J/s; $A$ – heat output of the engine – the amount of heat given by the engine to the cooling medium per unit of time at a temperature difference of 1 °C, J/(s°C); $\tau$ – the excess of the engine temperature above the temperature of the cooling medium, °C, is equal to:

$$\tau = \upsilon_{\text{en}} - \upsilon_{\text{c.m}}\,, \tag{19}$$

where $\upsilon_{\text{en}}$, $\upsilon_{\text{c.m}}$ are the temperature of the engine and the cooling medium, ºC, respectively; $C$ – the heat capacity of the engine – the amount of heat required to raise the engine temperature by 1ºC, J/ºC.

Dividing the terms of equation (18) by $Adt$, we get:

$$\frac{Q}{A} = \tau + \frac{C}{A}\frac{d\tau}{dt}\,, \tag{20}$$

or

$$\tau + T_{\text{nom}}\frac{d\tau}{dt} = \tau_y\,, \tag{21}$$

where $T_{\text{nom}}$ is the engine heating time constant – the nominal time during which the temperature rise from $\tau = 0$ would reach the value set by $\tau_y$ at $Q$ = const and the absence of heat transfer to the cooling medium, $T_{\text{nom}} = C/A$.

Solving equation (21):

$$\tau = \tau_y(1 - e^{-t/T_{\text{nom}}}) + \tau_0 e^{-t/T_{\text{nom}}}\,, \tag{22}$$

where $\tau_y$, $\tau_0$ are, respectively, the final (set) and initial value of the engine temperature exceeding the ambient temperature:

$$\tau_y = Q / A\,. \tag{23}$$

If $\tau_0 = 0$, then (22) takes the form:

$$\tau = \tau_y(1 - e^{-t/T_{\text{nom}}})\,. \tag{24}$$

In real conditions, as a result of the heat transfer of the engine during the time $T_{\text{nom}}$, the engine temperature will exceed the value $\tau = 0.632\tau_y$, which follows from (24), in which $t = T_{\text{nom}}$:

$$\tau = \tau_y(1 - e^{-1}) = 0.632\tau_y\,. \tag{25}$$

The actual heating curve is slightly different from the exponent. At the beginning of the heating process, the engine temperature increases faster than according to the theoretical curve, and only starting from $\tau = (0.5...0.6)\tau_y$ to $\tau = \tau_y$ the real curve approaches the exponential one.

In self-ventilated open engines of small and medium power, the time constant is about 1 hour, in closed engines of high power – 3...4 hours. When the self-ventilated engine is turned off and stopped, the cooling time constant $T_0$ turns out to be significantly greater than the heating $T_{\text{nom}}$. This is explained by the fact that when the self-ventilating engine stops, its heat output decreases. Coefficient of deterioration of heat transfer with a stationary rotor:

$$\beta_0 = A_0 / A\,, \tag{26}$$

where $A_0$, $A$ are the heat output, respectively, at a stationary engine and nominal angular speed. Approximate values of the coefficient $\beta_0$ for engines of various designs [14] are given in the Table 2.

It is clear that the presence and consideration of additional components related to heat exchange in the engine in the models makes the models more accurate and more suitable for calculations. However, the main disadvantage is the need for information about the numerous coefficients that appear in the matrices of heat capacities and thermal conductivities, so simplified models are often used in practice.

**Table 2**

Approximate values of the coefficient $\beta_0$

| Engine execution | $\beta_0$ |
|---|---|
| Closed with independent ventilation | 1 |
| Closed without forced cooling | 0.95...0.98 |
| Closed self-ventilated | 0.45...0.55 |
| Self-ventilated protected | 0.25...0.35 |

**Algorithm of IM model**

The model of an induction electric motor is made on the basis of the models of all elements included in its composition. The scheme of the IM simulation algorithm is presented in Fig. 1. The calculation performed on the basis of the specified algorithm allows simulating the start-up and operating mode of the established IM. This model can be used in known AC drives.

Currently, thyristor voltage converters are actively used as soft starter devices for high-power electric motors with a supply voltage of up to 1 kV, as well as for high-voltage asynchronous and synchronous motors with a voltage of 3, 6 and 10 kV. The operation of the model on the example of operation of IM from the thyristor voltage converter needs an explanation.

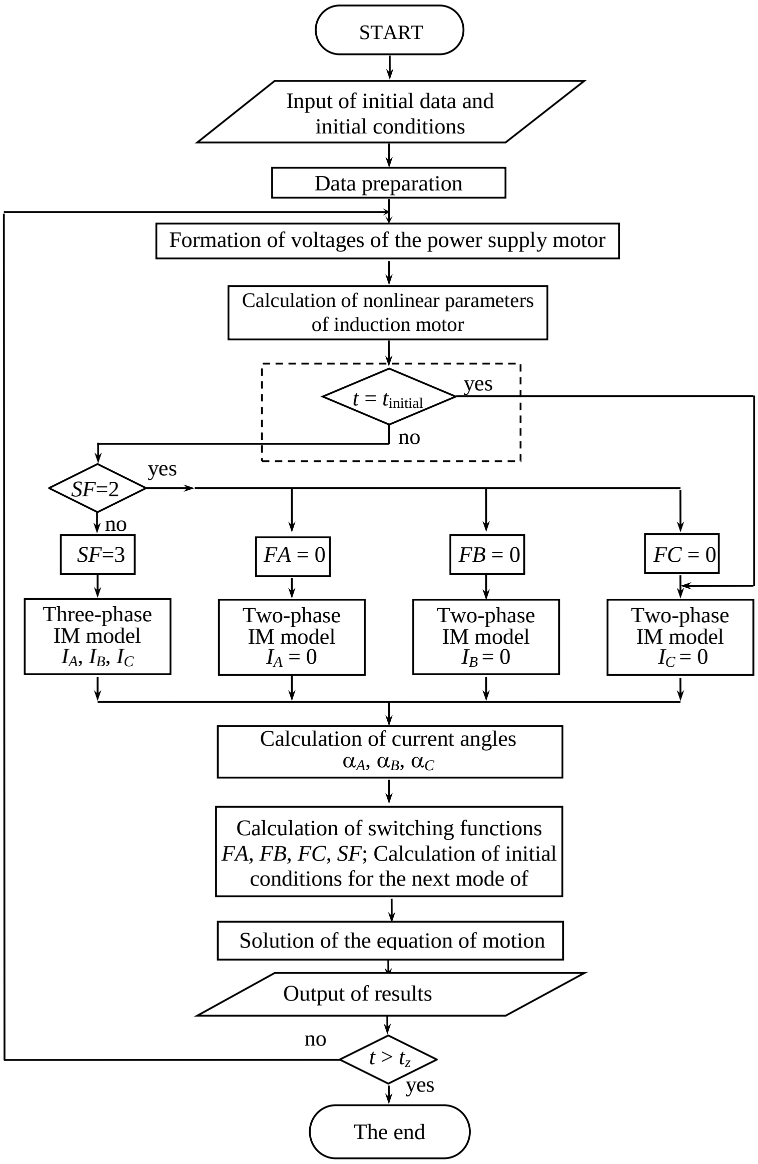


**Fig. 1.** Scheme of the IM simulation algorithm

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Thus, the input of output data requires the operator to assign all parameters of the thyristor voltage converter blocks of induction motors to the appropriate conditions and operating mode. Mandatory for the task are: initial electromagnetic conditions, engine parameters, total moment of inertia of the electric drive, magnitude and nature of the loIM moment, control angle, time of the specified cycle, etc.

The output data is prepared, typical examples of which are the recalculation of the parameters of induction motors from the *L*-shaped scheme to the *T*-shaped one, as well as the calculation of additional output data based on the input data. The formation of the three-phase voltage system is performed on the basis of the power source model. Further work of the model involves taking into account the nonlinearities of the parameters of the induction motor and starting it by connecting it to the power supply motor.

A dotted line on the diagram highlights the block of control of the initial conditions of inclusion. Depending on the conditions for the formation of IM operation modes, the transition to three- or two-phase models is carried out (logical-intellectual function *SF*, see Fig. 1). This causes the problem of matching the final electromagnetic conditions of three-phase switching with the initial conditions of the two-phase mode of operation of the induction motor. The transition is made under the condition of changing the sign of the current, which at the previous step of the calculation, although close, is not equal to zero, and at the next step, it is conditionally given a value equal to zero. The consequences of such misalignment are expressed in the appearance of spikes in the moment curve, which distort its real shape. In part, this problem can be solved by drastically reducing the integration step - h. However, based on the condition of continuity of flux linkages of non-disconnected phases, at any transition to the two-phase mode, it is necessary to find new values of the stator and rotor currents corresponding to the new circuit configuration. The further calculation involves determining the current angles $\alpha$ of the negative-torque voltage converters. After the calculation of the switching functions, the IM operating conditions should be fixed, which would be considered as the initial operating conditions for the next calculation cycle. After solving one of the numerical methods of integration of the equation of motion, the current value of the unknown function is presented in a form convenient for the operator. After that, the calculation cycle is repeated. When the calculated time *t* reaches the specified value of the cycle time $t_z$, the simulation process is completed.

**The results**

Verification of the adequacy of the developed mathematical model for the studied IM was carried out by comparing the results of simulation and experiment. The simulation results were performed for an electric motor with a capacity of 1.5 to 500 kW. As a result, the simulation confirms that the difference in the calculated data is within acceptable limits. The results of experimental and calculated data for some modes of operation of IM with a capacity of 447 kW and a voltage of 6 kV are shown in Fig. 2 – 7. The error in the currents does not exceed 5 %.

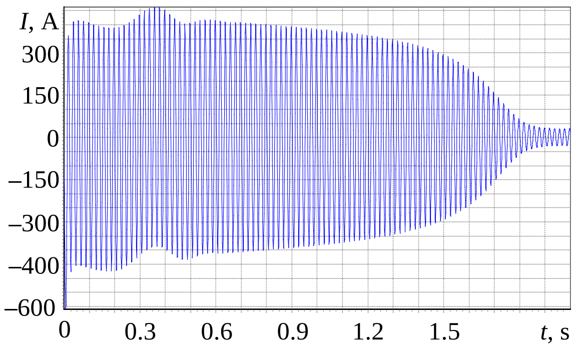


**Fig. 2.** The calculated curve of the change in the instantaneous values of the IM stator current with a capacity of 447 kW and a voltage of 6 kV during the idle start-up

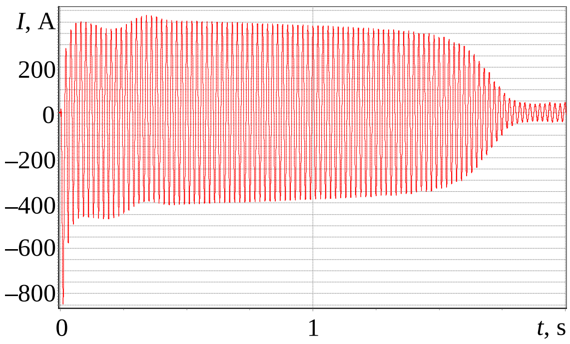


**Fig. 3.** Experimental curve of changes in the instantaneous values of the IM stator current with a power of 447 kW and a voltage of 6 kV during the idle start-up

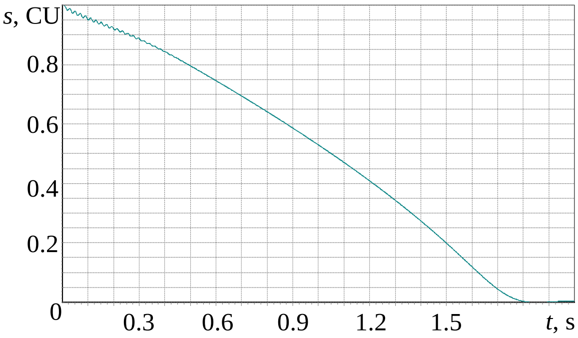


**Fig. 4.** The calculated curve of the slip change of IM with a capacity of 447 kW, a voltage of 6 kV at idle start-up

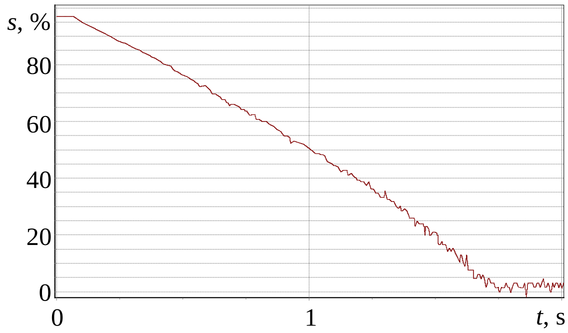


**Fig. 5.** Experimental curve of the slip change of IM with a capacity of 447 kW and a voltage of 6 kV at idle start-up

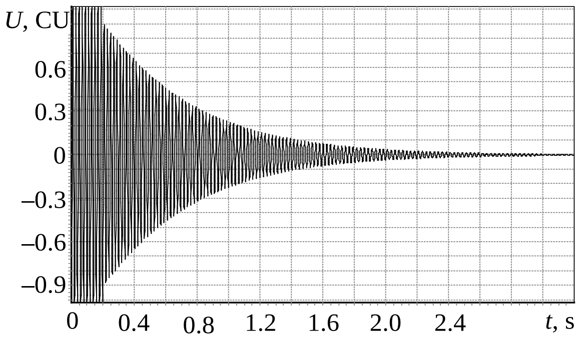


**Fig. 6.** Calculated curve of changes in the instantaneous relative values of the phase voltage of the IM stator winding with a power of 447 kW and a voltage of 6 kV at idle

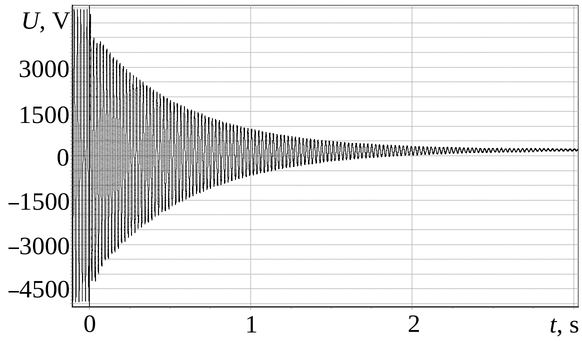


**Fig. 7.** Experimental curve of the change in the phase voltage of the IM stator winding with a power of 447 kW and a voltage of 6 kV at idle

## Conclusions

1. An improved mathematical model of an induction motor is proposed. The model uses the author's method of taking into account motor nonlinearities, in particular, the saturation of the magnetic circuit along the main path and dissipation paths, the effect of current displacement, and the main thermal nonlinearities.

2. The improved model, thanks to the use of natural three-phase axes *A*, *B*, *C*, a fixed coordinate system, makes it possible to consider the features of IM operation with the possibility of power supply from any known semiconductor converters.

3. The proposed model makes it possible to obtain results adequate to the electromechanical and electromagnetic processes of a real engine and to carry out the necessary studies of static and dynamic modes of operation of electromechanical systems. Differences between calculated and experimental data do not exceed 6...11 %, in static modes, and 17...22 %, in dynamic modes.

4. When performing the following studies, further improvement of the model is expected by using a more complex thermal model of IM. This will allow detailed consideration of the effect of the operating mode on engine heating, and the presence and consideration in the model of additional components related to heat exchange in the engine will make it even more accurate.

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