MACHINE BUILDING

МАШИНОБУДУВАННЯ

UDC 629.371.21

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CALCULATION OF THE STABILITY FLAT SHAPE BENDING OF THE RACING CAR FRAME STRUCTURAL ELEMENTS IN THE CIRCULAR ARCHES FORM

В.Ф. Оробей, О.М. Лимаренко, А.Ю. Бажанова, В.В. Хамрай, А.А. Пономаренко. **Розрахунок на стійкість плоскої форми згинання елементів конструкції рами гоночного автомобіля у вигляді кругових арок.** Для підвищення міцності та жорсткості характеристик шарнірні елементи конструктивних елементів гоночних автомобілів мають велике співвідношення осьових моментів інерції поперечних перерізів. В роботі отримана методика розв'язання крайових завдань стійкості плоскої форми згинання елементів конструкцій гоночних автомобілів у вигляді кругових арок з перерізами, що мають декілька осей симетрії. В автомобілях класу Формула ці елементі є найбільш відповідальними за безпеку життя і неушкодженість людини-пілота. В роботі виконано інтегрування системи двох диференціальних рівнянь стійкості зазначених конструктивних елементів рами гоночного автомобіля у вигляді кругових арок або криволінійних стержнів. Для дослідження використано чисельно-аналітичний метод граничних елементів розроблений професором Оробеєм В.Ф. В статті наведено два варіанти систем фундаментальних ортонормованих функцій для диференціальних рівнянь стійкості кругових арок з постійними коефіцієнтами які отримані в ході досліджень. Задачу стійкості конструктивних елементів гоночних автомобілів що за своєю геометрією відповідають круговим аркам вирішено числовим методом що набуває стрімкого розвитку, метод має теоретично доведені точні рішення. Отримане в ході досліджень рівняння може бути застосоване до вирішення дуже складних завдань стійкості різноманітних конструкцій, що містять стержні, окреслені по дузі кола. Рівняння можна застосовувати для рішень дуже складних задач стійкості різних конструкцій, що містять стержні, окреслені по дузі кола. Такі конструктивні елементи використовуються в багатьох конструкціях галузевого машинобудування.
Ключові слова: крайові завдання стійкості, система лінійних диференціальних рівнянь із змінними коефіцієнтами, фундаментальні ортонормовані функції, метод граничних елементів, гоночний автомобіль, формула, рама

V. Orobey, O. Lymarenko, A. Bazhanova, V. Khamray, A. Ponomarenko. **Calculation of the stability flat shape bending of the racing car frame structural elements in the circular arches form.** To increase the strength and rigidity of the characteristics, the articulated elements of structural racing cars have a large ratio of axial moments of inertia of the cross sections. The method of solving boundary value problems of stability of the flat form of bending of racing car structural elements in the form of circular arches with sections having several axes of symmetry is obtained. In Formula Class cars, these elements are most responsible for the safety of the pilot. The system of integration of two differential equations of stability of the specified constructive elements of a car racing frame in the form of circular arches or curvilinear cores is executed in work. The numerical-analytical method of limiting elements developed by Professor V.F. Orobey was used for the research. The article presents two variants of systems of fundamental orthonormal functions for differential equations of stability of circular arches with constant coefficients obtained during research. The problem of stability of structural elements of racing cars on the geometry corresponding to circular arches is solved by a numerical method acquiring rapid development; the method has theoretically proved exact decisions. The equation obtained in the course of research is applicable to the solution of very complex problems of stability of various structures containing rods delineated along the arc of a circle. The equations can be used to solve very complex problems of stability of various structures containing rods drawn along the arc of a circle. Such structural elements are used in many designs of industrial engineering.
Keywords: boundary value problems of stability, system of linear differential equations with variable coefficients, fundamental orthonormal functions, boundary element method, racing car, formula, frame

Introduction

In order to improve the strength and stiffness characteristics, the pivot elements of the structural elements of the race cars have a large ratio of the axial moments of inertia of the cross sections (Fig. 1).

DOI: 10.15276/opu.2.64.2021.01

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

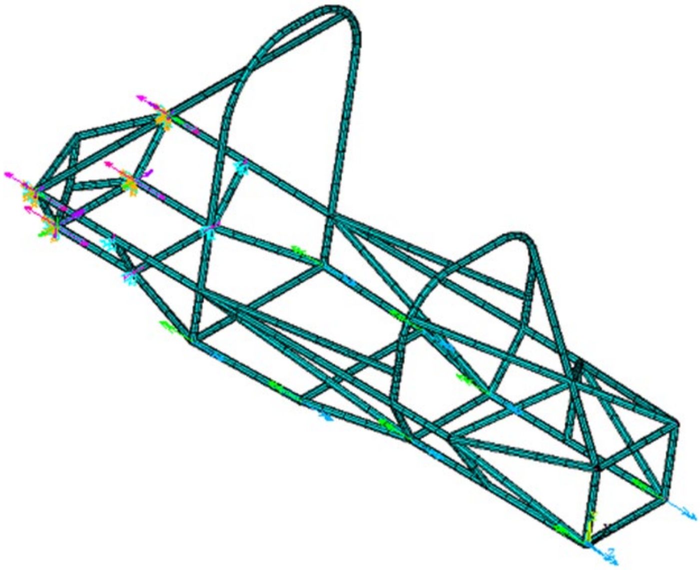

Fig. 1. Frame of a Formula racing car

In this case, the design of the space frame satisfies the conditions of strength and rigidity, but at the same time, there is a possibility of loss of stability of the flat form of bending. After buckling, the bar structural element experiences two bends and torsion. Variable cross-sections often lead to various breakdowns and the possibility of safe operation. Therefore, the problem of preventing such phenomena remains urgent.

Analysis of literary data and problem statement

The problem of the stability of the plane bending of rectilinear bars with sections in the form of a narrow strip was formulated in the 19th century. The outstanding mechanical scientist S.P. Timoshenko made a significant contribution to solving the problems of stability of thin-walled I-beams. Professor V.Z. Vlasov [1] generalized the theory of spatial stability of rod elements. Note that it was not possible to use the constructed theory for a long time, since the corresponding differential equations had variable coefficients and their integration ran into serious mathematical difficulties. This problem found its effective solution only with the development of a numerical-analytical version of the boundary element method (BEM), which allows mathematically rigorously and accurately solve boundary value problems for linear homogeneous and inhomogeneous differential equations with variable coefficients [2 – 5]. If for rectilinear rods various solutions of differential stability equations have been accumulated, then for curved rods - circular arches there are no fundamental functions-solutions of the Cauchy problems of stability of a plane bending form. Thus, the construction of a system of fundamental orthonormal functions for problems of stability of structural elements in the form of arches is an urgent problem.

Purpose and tasks of the research

The aim of the presented work is to construct a system of fundamental orthonormal functions for problems of stability of a plane bending form of structural elements of a racing car frame in the form of circular arches with sections with several axes of symmetry. This goal involves solving the problem of integrating the corresponding system of differential equations.

Development of a digital solution to the problem

The system of equations for the stability of a flat form of bending of a circular bar with sections having two or more axes of symmetry prof. V.Z. Vlasov is reduced to the form [1]:

$$\begin{cases} EI_y\omega^{IV}(\alpha) + \dfrac{EI_\omega}{R}\theta^{IV}(\alpha) + \left[M_z(\alpha) - \dfrac{GI_d}{R}\right]\theta^{II}(\alpha) = 0; \\ EI_\omega\theta^{IV}(\alpha) + GI_d\theta^{II}(\alpha) + \left[M_z(\alpha) - \dfrac{EI_y}{R}\right]\omega^{II}(\alpha) = 0, \end{cases} \quad (1)$$

where EI_y – horizontal bending stiffness xOz (Fig. 2); $\omega(\alpha)$ – bending movement of the bar axis along the axis Oz; EI_y – sectorial stiffness of the section under constrained torsion; R – radius of the axis of the circular bar; $\theta(\alpha)$ – angle of twisting of the section around the axis Ox; $M_z(\alpha)$ – bending moment in a section caused by a given shear load; GI_d – torsional stiffness; α – angular coordinate of the current section.

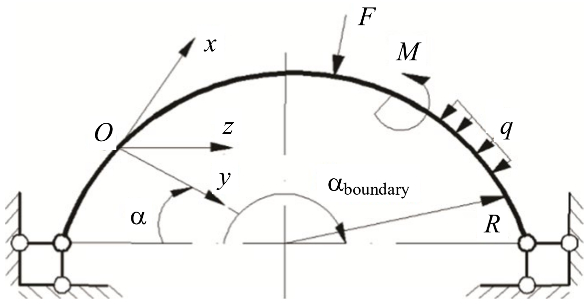

Fig. 2. Design scheme of the problem of stability of a circular bar

The task can be significantly simplified if we use the numerical-analytical version of the BEM [2 – 5]. In this method, you need to have a solution to the Cauchy problem for equation (1), but with constant coefficients. Let us outline the procedure for integrating a simplified system of equations. The initial parameters of constrained torsion and bending in the horizontal plane are as follows:

$GI_d\theta(0)$ – twist angle, kNm^2;

$GI_d\theta^{I}(0)$ – twist angle derivative, kNm^2;

$B_\omega(0) = -\dfrac{GI_d}{k^2}\theta^{II}(0)$ – bimoment, kNm^2;

$k = \sqrt{\dfrac{GI_d}{EI_\omega}}$ – bending-torsional characteristic of a section, 1/m;

$M_\omega(0) = -\dfrac{GI_d}{k^2}\theta^{III}(0)$ – flexural torque, kNm;

$EI_y\omega(0)$ – displacement of the section in the direction of the axis Oz, kNm^3;

$EI_y\omega'(0) = EI_y\varphi(0)$ – section rotation angle, kNm^2;

$EI_y\omega''(0) = -M_y(0)$ – horizontal bending moment, kNm;

$EI_y\omega'''(0) = -Q_z(0)$ – lateral force in the horizontal plane, kN.

These initial parameters and the system of equalizations (1) form the Cauchy problem of stability of the plane of the bending shape of a circular bar. To form fundamental solutions to the Cauchy problem, we will perform a number of transformations.

It follows from the second equation of system (1) that ($M_z = \text{const}$):

$$\omega''(\alpha) = \frac{1}{\left(M_z - \dfrac{EI_y}{R}\right)}[-EI_\omega\theta^{IV}(\alpha) + GI_d\theta^{II}(\alpha)]. \tag{2}$$

By integrating this expression twice, we obtain the relationship between the bending displacement $\omega(\alpha)$ and the twist angle $\theta(\alpha)$:

$$\omega(\alpha) = \frac{1}{\left(M_z - \dfrac{EI_y}{R}\right)}[-EI_\omega\theta^{II}(\alpha) + GI_d\theta(\alpha)] + (A\cdot\alpha + B)\frac{1}{\left(M_z - \dfrac{EI_y}{R}\right)}. \tag{3}$$

Here the integration constants are equal:

$$\begin{aligned} B &= \left(M_z - \frac{EI_y}{R}\right)\omega_{(0)} + EI_y\omega''(0) - GI_d\theta(0); \\ A &= \left(M_z - \frac{EI_y}{R}\right)\omega'_{(0)} + EI_\omega\theta'''(0) - GI_d\theta'(0). \end{aligned} \tag{4}$$

 ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

Substitute $w''_{(\alpha)}$ rom (2) to the first equation of system (1), then we obtain the resolving differential equation of stability of the plane form of bending of a circular bar:

$$-z_1\theta^{\mathrm{VI}}_{(\alpha)} + -z_2\theta^{\mathrm{IV}}_{(\alpha)} + -z_3\theta^{\mathrm{II}}_{(\alpha)} = 0, \tag{5}$$

where:

$$z_1 = \frac{EI_y \cdot EI_\omega}{\left(M_z - \dfrac{EI_y}{R}\right)};\; z_2 = \frac{EI_y \cdot GI_d}{\left(M_z - \dfrac{EI_y}{R}\right)} + \frac{EI_\omega}{R};\; z_3 = M_z - \frac{GI_d}{R}. \tag{6}$$

Equation (5) is classified as a linear homogeneous differential equation with constant sixth order coefficients. Its solution can be obtained according to the standard scheme. The characteristic equation for (5) has the form:

$$(-z_1)t^6 + z_2 t^4 + z_1 t^2 = 0. \tag{7}$$

Its roots are of different types. Let's look at two of the most important root combinations.

First case:

$t_{1,2=0}$ – real multiples;

$$z_{3,4} = \pm\sqrt{\frac{-z_2^2 + 4z_1z_3}{-2z_1}} \text{ two real roots;} \tag{8}$$

$$t_{5,6} = \pm i\sqrt{\frac{z_2 + \sqrt{z_2^2 + 4z_1z_3}}{2z_1}} \text{ two minimal roots.}$$

We write the general solution of equation (5) in the form:

$$\theta(\alpha) = C_1 + C_2\cdot\alpha + C_3\,\mathrm{ch}\,a\alpha + C_4\,\mathrm{sh}\,a\alpha + C_5\cdot\cos b\alpha + C_6\sin b\alpha\,, \tag{9}$$

where $a = \sqrt{\dfrac{-z_2 + \sqrt{z_2^2 + 4z_1z_3}}{-2z_1}}$. (10)

We differentiate expression (9) five times, taking into account the relationship between the initial parameters and expression (3), and for the constants of integration we compose a system of linear algebraic equations $C_1 - C_6$:

1	2	3	4	5	6			
1		1		1		C_1		$\theta_{(0)}$
	1		a		b	C_2		$\theta^{\mathrm{I}}_{(0)}$
		a^2		$-b^2$		C_3		$-\dfrac{B_{\omega(0)k^2}}{GI_d}$
			a^3		$-b^3$	C_4	=	$-\dfrac{M_{\omega(0)k^2}}{GI_d}$
		A_{53}		A_{55}		C_5		$-\dfrac{M_{y(0)}}{EI_y}$
			A_{64}		A_{66}	C_6		$-\dfrac{Q_{z(0)}}{EI_y}$

, (11)

where the elements of the coefficient matrix have the form:

$$A_{53}=\frac{a^2(-EI_\omega a^2+GI_d)}{M_z-\dfrac{EI_y}{R}};\quad A_{55}=\frac{b^2(EI_\omega b^2+GI_d)}{M_z-\dfrac{EI_y}{R}};$$
$$A_{64}=\frac{a^3(-EI_\omega a^2+GI_d)}{M_z-\dfrac{EI_y}{R}};\quad A_{66}=\frac{b^3(EI_\omega b^2+GI_d)}{M_z-\dfrac{EI_y}{R}}. \tag{12}$$

After solving the system of equations (11), the integration constants are written in the form:

$$C_2=\theta^{\mathrm{I}}_{(0)}-\frac{a^2+b^2}{x_1b^2-x_2a^2}\left[-\frac{Q_{z(0)}}{EI_y}\right]+\frac{x_1+x_2}{x_1b^2x_2a^2}\left[-\frac{M_{\omega(0)k^2}}{GI_d}\right];$$
$$C_3=\frac{b^2}{x_1b^2-x_2a^2}\left[-\frac{M_{y(0)}}{EI_y}\right]-\frac{x_2}{x_1b^2x_2a^2}\left[-\frac{B_{\omega(0)k^2}}{GI_d}\right];$$
$$C_4=\frac{b^2}{a(x_1b^2-x_2a^2)}\left[-\frac{Q_{z(0)}}{EI_y}\right]-\frac{x_2}{a(x_1b^2x_2a^2)}\left[-\frac{M_{\omega(0)k^2}}{GI_d}\right]; \tag{13}$$
$$C_5=\frac{a^2}{x_1b^2-x_2a^2}\left[-\frac{M_{y(0)}}{EI_y}\right]-\frac{x_1}{x_1b^2x_2a^2}\left[-\frac{B_{\omega(0)k^2}}{GI_d}\right];$$
$$C_6=\theta^{\mathrm{I}}_{(0)}-\frac{a^2}{b(x_1b^2-x_2a^2)}\left[-\frac{Q_{z(0)}}{EI_y}\right]+\frac{x_1}{b(x_1b^2x_2a^2)}\left[-\frac{M_{\omega(0)k^2}}{GI_d}\right],$$

where indicated:

$$x_1=\frac{a^2(-EI_\omega a^2+GI_d)}{M_z-\dfrac{EI_y}{R}};\quad x_2=\frac{b^2(EI_\omega b^2+GI_d)}{M_z-\dfrac{EI_y}{R}}. \tag{14}$$

The constants C_1-C_6 are substituted into the expression for the twist angle $\theta(\alpha)$ (9), and then four parameters of bending (using expression (3)) and four parameters of constrained torsion with respect to the corresponding initial parameters can be formed. After normalizing the fundamental functions, these expressions can be conveniently represented in matrix form as follows:

	=	1	2	3	4	5	6	7	8	
$EI_y\omega_{(\alpha)}$	1	1	α	$-A_{13}$	$-A_{14}$			$-A_{17}$	$-A_{18}$	$EI_y\omega_{(0)}$
$EI_y\varphi_{(\alpha)}$	2		1	$-A_{23}$	$-A_{24}$			$-A_{27}$	$-A_{28}$	$EI_y\varphi_{(0)}$
$M_{y(\alpha)}$	3			A_{33}	A_{34}			A_{37}	A_{38}	$M_{y(0)}$
$Q_{z(\alpha)}$	4			A_{43}	A_{44}			A_{47}	A_{48}	$Q_{z(0)}$
$GI_d\theta_{(\alpha)}$	5			$-A_{53}$	$-A_{54}$	1	α	$-A_{57}$	$-A_{58}$	$GI_d\theta_{(0)}$
$GI_d\theta'_{(\alpha)}$	6			$-A_{63}$	$-A_{64}$		1	$-A_{67}$	$-A_{68}$	$GI_d\theta'_{(0)}$
$B_{\omega(\alpha)}$	7			A_{73}	A_{74}			A_{77}	A_{78}	$B_{\omega(0)}$
$M_{\omega(\alpha)}$	8			A_{83}	A_{84}			A_{87}	A_{88}	$M_{\omega(0)}$

, (15)

where the fundamental orthonormal functions take the form:

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

$$
\begin{gathered}
A_{13}=\frac{-(a^2+b^2)c+b^2\dfrac{x_1}{a^2}\operatorname{ch}a\alpha+a^2\dfrac{x_2}{b^2}\cos b\alpha}{x_1b^2-x_2a^2};\quad c=\frac{GI_d}{M_z-\dfrac{EI_y}{R}};\\
A_{14}=\frac{-ab(a^2+b^2)ca+b^3\dfrac{x_1}{a^2}\operatorname{sh}a\alpha+a^3\dfrac{x_2}{b^2}\sin b\alpha}{ab(x_1b^2-x_2a^2)};\\
A_{17}=\frac{k^2(x_1+x_2)c-k^2x_2\dfrac{x_1}{a^2}\operatorname{ch}a\alpha+k^2x_1\dfrac{x_2}{b^2}\cos b\alpha+(x_1b^2-x_2a^2)ca}{x_1b^2-x_2a^2}\cdot\frac{EI_y}{GI_d};\\
A_{18}=\frac{k^2ab(x_1+x_2)ca-k^2x_2\dfrac{x_1}{a^2}\operatorname{sh}a\alpha-k^2ax_1\dfrac{x_2}{b^2}\sin b\alpha+ab(x_1b^2-x_2a^2)ca}{ab(x_1b^2-x_2a^2)}\cdot\frac{EI_y}{GI_d};\\
A_{23}=\frac{x_1b^3\operatorname{sh}a\alpha-x_2b^3\sin b\alpha}{ab(x_1b^2-x_2a^2)};\quad A_{24}=A_{13};\,A_{34}=A_{23};\\
A_{27}=\frac{-k^2x_1x_2b\operatorname{sh}a\alpha+k^2x_1x_2a\sin b\alpha}{ab(x_1b^2-x_2a^2)}\cdot\frac{EI_y}{GI_d};\quad A_{44}=A_{33};\\
A_{33}=\frac{x_1b^2\operatorname{ch}a\alpha-x_2a^2\cos b\alpha}{x_1b^2-x_2a^2};\quad A_{37}=\frac{[-x_1x_2(\operatorname{ch}a\alpha-\cos ba)]}{x_1b^2-x_2a^2}\cdot\frac{EI_y}{GI_d};\\
A_{43}=\frac{x_1ab^2\operatorname{sh}a\alpha+x_2a^2b\sin b\alpha}{x_1b^2-x_2a^2};\quad A_{47}=\frac{-x_1x_2(a\operatorname{sh}a\alpha-b\sin ba)k^2}{x_1b^2-x_2a^2}\cdot\frac{EI_y}{GI_d};\\
A_{28}=A_{17};\,A_{38}=A_{27};\quad A_{48}=A_{37};\quad A_{53}=\frac{-b^2(1-\operatorname{ch}a\alpha)-a^2(1-\cos ba)}{x_1b^2-x_2a^2}\cdot\frac{EI_y}{GI_d};\\
A_{54}=\frac{-b^3(a\alpha-\operatorname{sh}a\alpha)-a^3(ba-\sin\alpha)}{x_1b^2-x_2a^2}\cdot\frac{GI_d}{EI_y};\quad A_{57}=\frac{[x_2(1-\operatorname{ch}a\alpha)+x_1(1-\cos b\alpha)]k^2}{x_1b^2-x_2a^2};\\
A_{58}=\frac{[bx_2(a\alpha-\operatorname{sh}ba\alpha)+ax_1(ba-\sin b\alpha)]k^2}{ab(x_1b^2-x_2a^2)};\quad A_{63}=\frac{ab^2\operatorname{sh}a\alpha+a^2b\sin b\alpha}{x_1b^2-x_2a^2}\cdot\frac{GI_d}{EI_y};\\
A_{64}=A_{53};\quad A_{67}=\frac{(-x_2a\operatorname{sh}a\alpha+x_1b\sin b\alpha)k^2}{x_1b^2-x_2a^2}\cdot\frac{GI_d}{k^2EI_y};\quad A_{68}=A_{57};\\
A_{73}=\frac{a^2b^2(\operatorname{ch}a\alpha-\cos b\alpha)}{x_1b^2-x_2a^2}\cdot\frac{GI_d}{k^2EI_y};\quad A_{74}=\frac{A_{63}}{k^2};\quad A_{77}=\frac{-x_2a^2\operatorname{ch}a\alpha+x_1b^2\cos b\alpha}{x_1b^2-x_2a^2};\\
A_{78}=\frac{A_{67}}{k^2};\quad A_{83}=\frac{a^2b^2\operatorname{sh}a\alpha+a^2b^3\sin b\alpha}{x_1b^2-x_2a^2}\cdot\frac{GI_d}{k^2EI_y};\\
A_{84}=A_{73};\quad A_{87}=\frac{-x_2a^3\operatorname{sh}a\alpha-x_1b^3\sin b\alpha}{x_1b^2-x_2a^2};\\
A_{88}=A_{77}.
\end{gathered}
\tag{16}
$$

Expression (15) is the resolving equation he BEM for solving boundary value problems of stability of a flat form of bending of a wide variety of structures in the form of individual arches, rings, ring systems and combined arch systems.

Discussion of the proposed approach to solving stability problems

Case. $M_z = \text{const}$.

This case for circular arches is very rare and is possible only when the supports are hinged and loaded with concentrated equal bending moments. In this case, equation (15) can be used directly for the entire structure according to the BEM algorithm [2 – 7].

The case when M_z there is some function of the angular coordinate.

For arched structures, this is the most common case. Here you need to have an analytical expression of the function $M_z(\alpha)$. The simplest way to construct this function is, again, according to the BEM algorithm [5, 6], where the procedure for calculating the function $M_z(\alpha)$ of existing loads is described in full detail. Then the arch is divided into n parts. In each part, the values M_z of the bending moment are calculated according to the known expression so that the area of M_z the stepped figure is equal to the area of M_z the actual diagram. If this condition is met, then at $n \geq 30$ practically accurate results of critical loads are obtained M_{tors}, F_{tors}, q_{tors} [2 – 5].

Conclusions

The analysis of the presented material shows that it is possible, within the framework of the algorithm of the numerical-analytical version of the BEM, to construct a resolving equation for the stability problems of the plane bending of circular rods, which are used as structural elements of racing cars. This equation can be applied to the solution of very complex problems of stability of various structures containing rods, outlined along an arc of a circle.

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Received September 18, 2021

Accepted November 11, 2021