MACHINE BUILDING

МАШИНОБУДУВАННЯ

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TECHNOLOGICAL DYNAMICS OF NON-STATIONARY SYSTEMS DURING FINISHING INTERMITTENT CUTTING

Г. Баланюк, О. Оргіян, Г. Оборський, В. Хобін. **Технологічна динаміка нестаціонарних систем при фінішному переривчастому різанні.** В роботі вивчено сталість та особливості коливань нестаціонарних технологічних систем при чистовому фінішному розточуванні у складних режимах різання – обробка переривчастих поверхонь або глибоких отворів малого діаметра тощо. У технології машинобудування такі операції виконуються досить часто, причому з постійно зростаючими вимогами щодо точності обробки. Зрозуміло, що в першому випадку перехідні процеси врізання і виходу інструменту, що періодично повторюються, викликають ударні впливи на різець, що призводить до сколювання різальних кромок, підвищеному зносу і негативно впливає на вихідну точність обробки. Замкнена на процес різання пружно-дисипативно-інерційна система стає нестаціонарною не тільки через переривання зв'язків, а й через періодичну зміну параметрів. В цьому випадку динамічні моделі описуються диференціальними рівняннями зі змінними коефіцієнтами. Системи з параметрами, що періодично змінюються, називають нестаціонарними, а коливання - параметричними. В сучасній технології машинобудування виникає багато завдань, у яких домінують динамічні чинники. Параметричні коливання описуються рівняннями Матьє, які відображають складні динамічні процеси, такі як резонанси та автоколивання. На стенді виконані експерименти з вивчення коливань при розточуванні зразків зі сталі, чавуну та бронзи з переривчастою поверхнею, причому кількість переривань за один оберт змінювалася від 1 до 20. Встановлено характер коливань та відображено умови сталості рішень на діаграмі Айнса-Стретта. Розроблено методику побудови часових форм коливань, що дозволяє прогнозувати амплітуди, частоти та резонансні явища при переривчастому різанні.
Ключові слова: нестаціонарна технологічна система, переривчасте різання, параметричні коливання, сталість, резонанс

A. Balaniuk, O. Orgiyan, G. Oborsky, V. Khobin. **Technological dynamics of non-stationary systems during finishing intermittent cutting.** In this work the stability and peculiarities of oscillations of unsteady technological systems during finishing boring in complex cutting modes - machining of discontinuous surfaces or deep holes of small diameter, etc. are studied. In mechanical engineering technology such operations are performed quite often, and with ever-increasing requirements for machining accuracy. It is clear that in the first case periodically repeated transient processes of tool plunging and tool exit cause impact effects on the cutter, which leads to chipping of cutting edges, increased wear and negatively affects the output machining accuracy. The elastic-dissipative-inertial system (EDIS) closed to the cutting process becomes unsteady not only due to the interruption of links, but also due to the periodic change of parameters. In this case, dynamic models are described by differential equations with variable coefficients. Systems with periodically changing parameters are called nonstationary, and oscillations are called parametric. In modern engineering technology, many problems arise in which dynamic factors dominate. Parametric oscillations are described by Mathieu equations, which reflect complex dynamic processes such as resonances and auto oscillations. Experiments were carried out on the bench to study oscillations during boring of steel, cast iron and bronze specimens with interrupted surfaces, with the number of interruptions per revolution varying from 1 to 20. The character of oscillations is established and the conditions of stability of solutions on the Ains-Strett diagram are reflected. A methodology of constructing time forms of oscillations has been developed, which makes it possible to predict amplitudes, frequencies and resonance phenomena in intermittent cutting.
Keywords: unsteady technological system, intermittent cutting, parametric oscillations, stability, resonance

Introduction

Unsteady dynamic systems are systems in which the parameters and characteristics change over time. Non-stationarity can be manifested by changes in external loads, cutting forces and modes, stiffness, mass, damping and other factors. The dynamics of non-stationary systems differs from stationary systems in that their behavior can vary significantly depending on time or operating conditions, as well as on the type of technological operation. A non-stationary system is described by differential equations in which the coefficients depend on time.

In problems of dynamics of mechanisms and machines, parametric oscillations occur quite often, for example, in the rotation of a shaft with different principal values of bending stiffness, in gears (due to the variable state of meshing of wheels), in various mechanisms where the configuration of the system periodi-

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cally changes. Such problems lead to the Mathieu-Hill equations, the properties of solutions of which determine the conditions of excitation of parametric oscillations: instability of unperturbed motions at some values of parameters of the elastic system. We obtain relations for determining the boundaries of instability regions, amplitudes of oscillations and characteristics of parameter variability at which the dynamic system cannot be described as stationary. These relations are presented in a criterial dimensionless form and form the basis for the calculation of parametric oscillations of technological systems.

The work reflects a new interdisciplinary area – process dynamics – which focuses on the study of vibrations, stability and their influence on accuracy parameters. This direction allows to integrate the results of analyses of machine dynamics with technological requirements for machining, which is especially important in high-precision manufacturing.

Analysis of recent publications and problem statement

The paper describes a cutting tool with a boring mandrel for measuring cutting force and vibration [1]. Sensors and electronics are embedded in the boring mandrel and measure vibration parameters in real time during cutting and transmit to control programs. In this work, the vibration parameters are correlated with machining quality. In [2], measurements of the friction coefficient in orthogonal cutting were made, reflecting the possibility of reducing vibrations and changes in the friction coefficient during the cutting process. Different designs of boring mandrels are compared in [3]. The characteristic features of oscillations and their corresponding spectral characteristics for different types of cutting were established. In works [4, 5] the methods of reducing the level of parametric oscillations using built-in vibration absorbers are given, and analytical methods for calculating oscillations of systems with variable stiffness are considered. In the same works and in [6], vibrations with stiffness variation according to the harmonic law are studied mainly. In [7], using the method of stitching of solutions taking into account the conditions of periodicity, graphs of stability regions are given. The stability of solutions of the dynamic model was calculated when changing the logarithmic decrement of oscillations, the parametric excitation coefficient and changes in the elastic characteristics of the technological system. In [8], the stability regions of parametric oscillations by classical Mathieu equations were calculated and given. Figure 1 shows the stability diagram for the Mathieu equation in general form:

$$\ddot{y}(x) + (a - 2q\cos(2x))y(x) = 0, \tag{1}$$

where $y(x)$ is the desired function;

$\ddot{y}(x)$ – second derivative of the function $y(x)$ on the variable *x*;

a and *q* are constant parameters that determine the specific type of equation.

It shows the areas of stable and unstable solutions. The stable regions are marked by shading, while the unstable ones remain white. This diagram illustrates the alternation of stable and unstable zones depending on the parameters *a* and *q* (coefficient a reflects amplitudes, and *q* – frequencies in the Mathieu special functions).

In [9, 10, 11] analytical solutions of the Mathieu equation using Runge-Kutt method [12, 13, 14] are given. When using nonlinear characteristics to suppress parametric oscillations, researchers often apply methods of linearization, harmonic balance, etc. [15, 16, 17].

Based on the analysis of published works on the peculiarities of discontinuous cutting, the aim and objectives of the research are formulated.

The aim is to increase and ensure the accuracy of fine boring of discontinuous surfaces.

Objectives:

1. To study the features of parametric vibrations of the technological system spindle-boring bar at intermittent boring;

2. Investigate the parametric instability of technological systems under piecewise constant perturbations – stiffness and cutting coefficient;

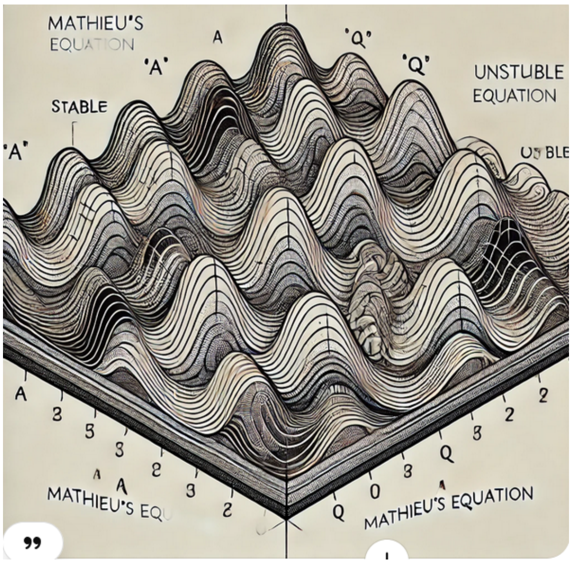

Fig. 1. Stability diagram for the Mathieu equation [8]

3. Model temporal waveforms to predict the occurrence of resonances and stable processing.

Statement of the main material and research results

Experimental study of vibrations at finishing boring of holes with discontinuous surface was carried out on the stand assembled on the basis of finishing-boring machine 2706B. The boring bar was set in such a way that the components P_y and P_z of the cutting force were orientated in the directions of the main stiffness axes of the spindle-boring bar elastic system. Samples from steel 45 and cast iron СЧ18 (SCh18) with the number of grooves j=1, 6 and 20 were bored (Fig. 2). The average value of the machined hole diameter was 40 mm. Boring of specimens was carried out by boring bars with diameter d=27 mm with l/d=3 and 4 (l – length of boring bar). The cutting speed was varied from 100 to 200 m/min in steps of 5 m/min at a constant feed rate of 0.025 mm/turn and two cutting depths of 0.05 and 0.1 mm.

Fig. 2. Samples to be processed

Typical experimental results are shown in Figure 3.

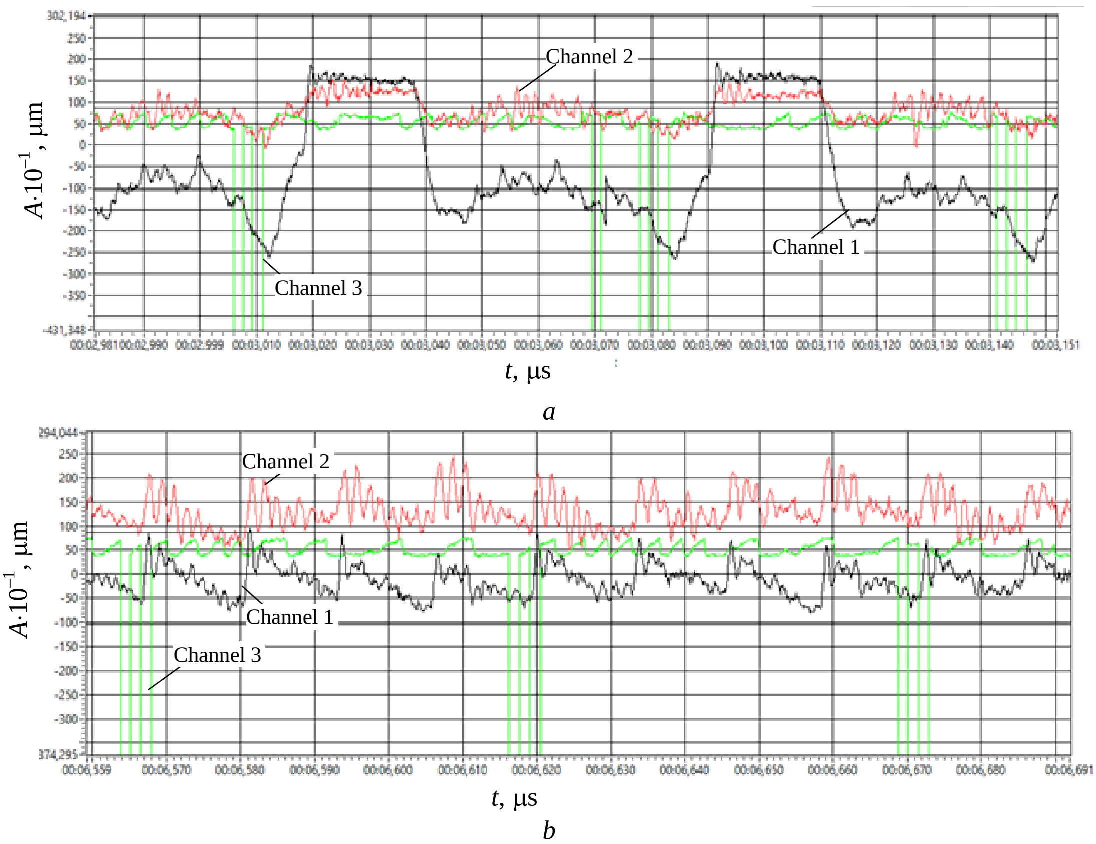

Fig. 3. Oscillograms of oscillations during machining of samples from steel 45: *a* – one groove, n=1000 min^{-1}, t=0.05 mm; *b* – 4 grooves, n=1200 min^{-1}, t=0.05 mm; (black line Y coordinate, red line Z coordinate)

It should be noted that immediately after plunging, the oscillations are decremental even under machining conditions that, when boring continuous surfaces, lead to the excitation of auto-oscillations. This is explained by the synchronizing effect of shock pulses during plunging, due to which the oscillations at two consecutive revolutions are almost in-phase, and the trace cutting mechanism stabilizes the closed dynamic system. As the cutting depth increases, the displacement of the equilibrium position (cutter pushback) increases and the initial values of the amplitudes of the high-frequency oscillations increase.

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The maximum values of vibration amplitudes when boring a discontinuous surface are 2...4 times greater than when boring a continuous surface, and the minimum values are the same as when there are no discontinuities.

Since the dynamics of intermittent cutting is determined by the ratio of the natural frequency of the system to the perturbation frequency, the dependence of the oscillation amplitude on the stiffness and mass of the UDIS is also non-monotonic. If the cutting speed control is somehow irrational, it is possible to remove the dynamic system from the dangerous resonance region by changing the natural frequency, for example, by increasing or decreasing the mass of the free end of the cantilevered boring bar.

A model with one degree of freedom is used to construct the time shapes. The equations have the form:

$$m\ddot{y} + b\dot{y} + cy = 0 - \text{no cutting}$$
$$m\ddot{y} + b\dot{y} + (c + k_c)y = k_c W - \text{cutting}, \qquad (2)$$

Or in general terms:

$$m\ddot{y} + b\dot{y} + (c + k_c \Phi(t))y = k_c W \Phi(t);$$
$$\Phi(t) = \begin{cases} 0 - iT_p \prec t \prec iT_p + \Delta t; \\ 1 - iT_p + \Delta t \prec t \prec (i+1)T_p, \end{cases} \qquad (3)$$

where T_p is the perturbation period;

i=0, 1, 2, 3 –number of revolutions;

$T_p = 2\pi / \omega$ – disturbance period;

$\omega_p = 2\pi f_p$, where f_p — is the linear excitation frequency, Hz;

ω_p — is the circular excitation frequency, s^{-1};

$f_p = \dfrac{\omega_p}{2\pi}$, $f_p = \dfrac{n}{60}$, n – min^{-1}.

Figure 4 shows the calculated time shapes of the oscillations.

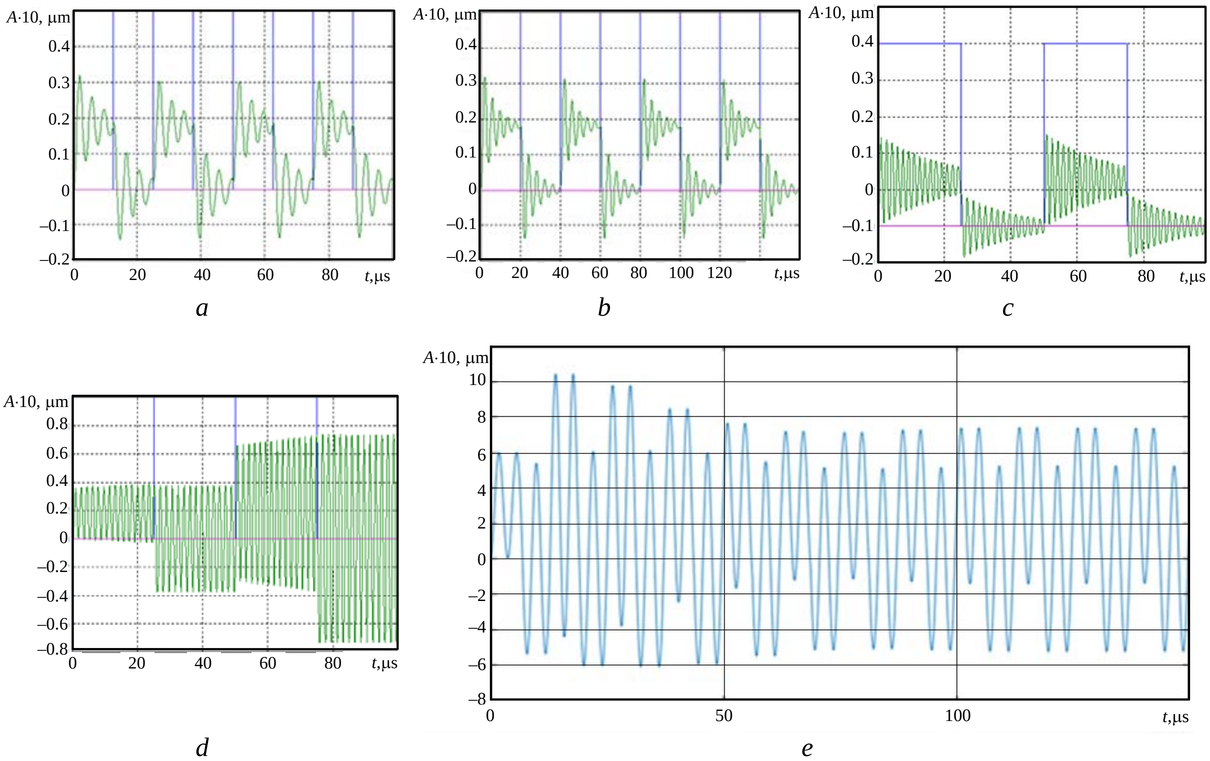

Fig. 4. Oscillograms of oscillations: *a* –2 grooves, *n*=1200 min^{-1}, *t*=0.05 mm, *b*=0.65 – attenuation; *b* – 2 grooves, *n*=750 min^{-1}, *t*=0.05 mm, *b*=0.65 – attenuation; *c* –1 groove and 1 protrusion, *n*=1200 min^{-1}, *t*=0.05 mm, *b*=0.65 attenuation; *d* – 1 groove and 1 tab, *n*=1200 min^{-1}; *t*=0.01 mm; *e* –6 grooves; *n*=800 min^{-1}; *t*=0.05 mm

The calculation was carried out in the Simulink software simulation environment integrated into the MATLAB universal system of computer mathematics [18, 19]. It was found that the calculated time forms of oscillations correspond to the experimental ones. The character of oscillations was established: high-frequency oscillations are also excited at the background of periodic changes in the equilibrium state at the moments of plunging into the ledge and the cutter exit into the slot. The decrement of oscillations during cutting differs from that measured during the slot passage and can be expressed through the parameters of the closed dynamic system.

It is known that parametric instability occurs with decreasing stiffness C and decrement of oscillations δ of the elastic system, as well as with increasing cutting coefficient k_p. In a stationary system, loss of stability is possible with the same change of parameters. It is of interest to ask the question about the significance of the influence of parametric perturbation on the stability of a closed dynamic system, which can be formulated as follows: whether it is possible to occur parametric instability in a stationary closed dynamic system, at values of its parameters that ensure stability. The answer to this question is given by comparing the stability conditions for stationary and nonstationary closed-loop systems.

To study the conditions of instability, let us write down the equations of the elastic system closed to the cutting process:

$$\begin{aligned} T_1^2\ddot{y} + T_2\dot{y} + y &= kP_z\Phi(t) \\ T_c\dot{P}_z + P_z &= K_c y\Phi(t) \end{aligned}, \qquad (4)$$

where $\Phi(t)$ is a function of time period $2\pi/\omega$, defined by the relations,

$$\Phi(t) = \begin{cases} 0 \text{ at } (i-1) \prec t \prec (i-1+g)T; \\ 1 \text{ at } (i-1+g)T \prec t \prec iT, \end{cases} \quad i = 1, 2...,$$

where $T_1 = 1/P$ is the inertial time constant, sec, P is the natural frequency of the elastic system, sec^{-1};

$T_2 = 2T_1^2 h$ – damping time constant, sec;

h – damping coefficient;

$g = l_s/(l_s + l_p)$ – form of parametric perturbation (l_s – slot width, l_p – protrusion width);

K_c – cutting coefficient, N/μm;

$T = 2\pi/\omega$ – perturbation period, sec;

ω – perturbation frequency sec^{-1};

T_c – chip formation time constant, sec;

k – is the coefficient of pliability, μm/N;

P_z – main component of cutting force, N;

y – horizontal movement of the cutter in the direction of the radial component of the cutting force.

The initial conditions for the solution of equation (4) are determined by the values of the cutting interruption function.

Let us transform the system of equations (4) at $\Phi(t) = 1$. Differentiating the first equation of the system and substituting it into the second equation, we obtain:

$$\frac{T_c}{k}(T_1^2\dddot{y} + T_2\ddot{y} + \dot{y}) + \frac{1}{k}(T_1^2\ddot{y} + T_2\dot{y} + y) = -K_c y\,.$$

We collect the coefficients at different powers of y and denote by:

$$k \cdot K_c = \gamma\,,$$

where γ is the dimensionless coefficient of coupling of the cutting process with the elastic system.

Let’s find:

$$T_cT_1^2\dddot{y} + (T_cT_2 + T_1^2)\ddot{y} + (T_c + T_2)\dot{y} + (1+\gamma)y = 0\,.$$

Let us write this equation in the form:

$$T_1^2\ddot{y} + T_2\dot{y} + (1+\gamma)y = -(T_cT_1^2\dddot{y} + T_cT_2\ddot{y} + T_c\dot{y})\,. \qquad (5)$$

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Considering that the solution of equation (5) describes oscillations whose frequency is close to the natural frequency of the elastic system, we require that:

$$T_c T_1^2 \dddot{y} + T_c \dot{y} = 0 .$$

Integrating this expression, we obtain:

$$T_c T_1^2 \ddot{y} = -T_c y .$$

Thus, the right-hand side of equation (5) is approximately equal to:

$$T_c T_1^2 \ddot{y} \approx -T_c T_2 \frac{y}{T_1^2} ,$$

and the equation will take the form:

$$T_1^2 \ddot{y} + T_2 \dot{y} + (1 + \gamma - \frac{T_c T_2}{T_1^2}) y \approx 0 .$$

Consequently, the interaction of an elastic system with a periodically interrupted cutting process in the above approximation is described by the equations:

$$\begin{gathered} T_1^2 \ddot{y} + T_2 \dot{y} + y = 0;\ \ 0 \prec t \prec q\frac{2\pi}{\omega}; \\ T_1^2 \ddot{y} + T_2 \dot{y} + \left(1 + \gamma - \frac{T_c T_2}{T_1^2}\right) y = 0;\ \ g\frac{2\pi}{\omega} \prec t \prec \frac{2\pi}{\omega}. \end{gathered} \tag{6}$$

The solution of these equations at the corresponding time intervals has the form:

$$\begin{gathered} y_1 = e^{-ht}(C_1 \cos p_1 t + C_2 \sin p_1 t); \\ y_2 = e^{-ht}(B_2 \cos p_2 t + B_2 \sin p_2 t), \end{gathered}$$

where

$$h = \frac{T_2}{2T_1^2};\ p_1 = \frac{1}{T_1};\ p_2 = \frac{\sqrt{1 + \gamma - \frac{T_c T_2}{T_1^2}}}{T_1},$$

and the constants C_1, C_2, B_1 and B_2 are determined from the initial conditions. The parametric stability of the solutions is investigated by the method of cross-linking taking into account the periodicity conditions:

$$\begin{gathered} y_1\left(q\frac{2\pi}{\omega}\right) = y_2\left(q\frac{2\pi}{\omega}\right);\quad y_2 = \left(\frac{2\pi}{\omega}\right) = Sy_1(0); \\ \dot{y}_1\left(q\frac{2\pi}{\omega}\right) = \dot{y}_2\left(q\frac{2\pi}{\omega}\right);\quad \dot{y}_2 = \left(\frac{2\pi}{\omega}\right) = S\dot{y}_1(0). \end{gathered}$$

The character of the system motion is determined by the value of S [20]. If $|S| \prec 1$, the displacements will gradually decrease and the system is stable. If $|S| \succ 1$, the displacements will increase, and in this case – the system is not stable.

Substituting the solutions of equations (5) into these conditions, we form the determinant, which must be equal to 0 to obtain solutions different from zero. Expanding this determinant, we obtain the characteristic equation:

$$z^2 - 2Az + 1 = 0,$$

where

$$A = \cos\alpha_1 \cos\alpha_2 - \frac{p_1^2 + p_2^2}{2p_1 p_2} \sin\alpha_1 \sin\alpha_2;$$

$$\alpha_1 = \frac{p_1 2\pi g}{\omega};\quad \alpha_1 = \frac{p_2 2\pi(1-g)}{\omega};\quad z = e^{h\frac{2\pi}{\omega}};$$

$$|S| = |z| e^{-h\frac{2\pi}{\omega}} \prec 1; \quad |z| \prec e^{h\frac{2\pi}{\omega}}.$$

Considering that:

$$z = A \pm \sqrt{A^2 - 1},$$

and after transformations we obtain that at the stability boundaries:

$$A^* = \mathrm{ch}\left(h\frac{2\pi}{\omega}\right) = \mathrm{ch}\left(\delta\frac{p_1}{\omega}\right).$$

We will carry out further reasoning for the main region of instability P/ω=0.5 and for g=0.5.

After some transformations we find the parametric stability condition:

$$\gamma \prec 2\frac{T_2}{T_1} + \frac{T_c T_2}{T_1^2}.$$

The stability condition of a closed dynamic system with constant coefficients is as follows:

$$\gamma \prec \frac{T_2}{T_p} + \frac{T_2^2}{T_1^2} + \frac{T_c T_2}{T_1^2}.$$

Comparing these two conditions, we conclude that in a closed dynamic system a parametric loss of stability is possible if the critical value γ , at which the parametric instability occurs, is less than the critical value for a stationary system:

$$T_c \prec \frac{T_1^2}{2T_1 - T_2}.$$

The characteristic value of the logarithmic decrement of oscillations for finishing boring machines is of the order of 0.1...0.2 and therefore $T_2 \neq T_1$. The obtained inequality shows that parametric resonance is possible in systems having sufficiently small natural frequencies (large T_1). Since the time constant of chip formation T_c decreases with increasing cutting speed ν, parametric instability can occur with increasing ν while other system parameters remain unchanged.

Conclusions

A new trend in mechanical engineering technology – process dynamics - allows us to focus on the study of vibrations, stability and their influence on precision parameters. This area allows the results of analyses of machine dynamics to be integrated with the technological requirements for machining, which is particularly important in high-precision production.

Fluctuations of closed non-stationary systems in technological dynamics are described most often by differential equations with variable coefficients, and the stability conditions of mechanical systems are described by dimensionless criteria.

The character of oscillations of the spindle-boring bar system under conditions of intermittent boring has been established. The oscillations have a pronounced decremental character with a periodic change of equilibrium states at the moments of plunge and exit of the cutter from the cutting zone. Thus, forced oscillations with a frequency higher than the free oscillations in the groove are excited during cutting, arising with a frequency equal to the natural frequency of the elastic system.

Time forms of oscillations obtained by modelling in Simulink simulation software integrated into MATLAB universal system of computer mathematics correspond to experimental oscillograms during boring of holes with discontinuous surface. Calculations of oscillations by time forms will allow predicting steady states of the technological system at given cutting modes.

Vibration amplitudes of cantilevered boring bars change significantly when the cutting speed is varied. This is due to changes in the frequency of excitation of parametric oscillations (ω), which are realized at the ratio of natural frequency P to the perturbation frequency ω equal to 0.5, 1, 1.5, 2. In order to avoid resonant values of the ratios P/ω , it is possible to change also the natural frequencies P, by designing the cantilevered boring bar with the possibility of changing its mass, for example by making a cavity in front of the cutting blade.

It is shown that in a stable stationary closed-loop system parametric instability is possible at increasing cutting speed under conditions of sufficiently small eigenfrequencies.

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