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O. Vatrenko, PhD, Prof.
Odesa National University of Technology, Kanatnaya Str. 112, Odesa, Ukraine, 65039; e-mail: alexvatrenko@gmail.com

LOSS OF STABILITY AND ENERGY OF METALLIC VACUUM MEMBRANES OF SEALING AGENTS

О. Ватренко. **Втрата стійкості та енергетика металевих вакуумних мембран закупорювальних засобів.** Деякі технічні системи функціонують за наявності вакууму. Для контролю глибини вакууму часто використовують датчики контролю вакууму, які є досить складними й недешевими електричними пристроями. В статті досліджуються енергетичні спроможності металевих мембран вакуумних закупорювальних засобів, які в процесі контрольованої втрати стійкості можуть виконувати функцію вакуумних датчиків в упаковці не застосовуючи спеціальні датчики. Дослідження показали, що мембрани, відштамповані на полі закупорювальних засобів, втрачають стійкість від перепаду тиску, а відновлення форми відбувається завдяки енергії трансформації, яка виникає в них в результаті втрати стійкості. Величина цієї енергії залежить від товщини мембрани. Отримано параметри роботи мембран різної товщини жерсті та побудовано відповідні графічні залежності. Згідно досліджень, зі зменшенням товщини мембрани енергія трансформації зростає, разом з тим тиск втрати стійкості зменшується. В процесі цифрового моделювання пропонується оцінювати рівень внутрішньої енергії металевих мембран порівнюючи енергетичні рівні різних мембран в можливих станах рівноваги. В основу пропонованого методу покладено математичне моделювання енергетичних рівнів стану рівноваги мембран, виконане з використанням енергетичного методу з теорії пластин та оболонок. Отримані енергетичні рівні мембран різної товщини жерсті у стані втрати стійкості, кожний з яких має свій власний мінімум. За результатами цифрового моделювання виконано аналіз енергетичної складової та розраховано мінімальні енергетичні показники під час втрати стійкості мембран для мембран різної товщини. Показано, що під час переходу з одного стану рівноваги в інший відбувається енергетичний стрибок. Під час енергетичного стрибка завдяки вивільненню енергії трансформації відбувається миттєве переміщення мембрани в просторі. Пропонується оцінювати енергетичні рівні металевих мембран методом, згідно якого порівнюються енергетичні рівні різних мембран в станах рівноваги.
Ключові слова: стійкість, прогин, енергетичний рівень, мембрана, втрата стійкості, рівновага

A. Vatrenko. **Loss of stability and energy of metallic vacuum membranes of sealing agents.** Some technical systems operate in the presence of vacuum. To control the vacuum depth, vacuum control sensors are often used, which are rather complex and expensive electrical devices. The paper investigates the energy capabilities of metal membranes of vacuum closures, which, in the process of controlled loss of stability, can perform the function of vacuum sensors in packaging without the use of special sensors. Studies have shown that membranes stamped on the field of closures lose their stability due to pressure drop, and the shape is restored due to the transformation energy that occurs in them as a result of the loss of stability. The value of this energy depends on the thickness of the membrane. The parameters of the membranes of different tinplate thicknesses were obtained and the corresponding graphical dependencies were constructed. According to the research, with a decrease in the membrane thickness, the transformation energy increases, while the pressure of loss of stability decreases. In the process of digital modelling, it is proposed to estimate the level of internal energy of metal membranes by comparing the energy levels of different membranes in possible equilibrium states. The proposed method is based on the mathematical modelling of the energy levels of the membrane equilibrium state, performed using the energy method from the theory of plates and shells. The energy levels of membranes of different tinplate thicknesses in the state of loss of stability are obtained, each of which has its own minimum. Based on the results of digital modelling, the energy component was analysed and the minimum energy values during the loss of membrane stability were calculated for membranes of different thicknesses. It is shown that an energy jump occurs during the transition from one equilibrium state to another. During the energy jump, due to the release of the transformation energy, the membrane moves instantaneously in space. It is proposed to estimate the energy levels of metal membranes by the method, according to which the energy levels of different membranes in equilibrium states are compared.
Keywords: stability, deflection, energy level, membrane, loss of stability, equilibrium

1. Introduction

Energy efficiency is one of the key areas of development of modern equipment and technologies. It implies the fullest possible use of the energy capabilities of any technical system or technological process. One of the most common methods used in the food industry to preserve the quality of long-term storage products and extend their shelf life is packaging and storage under vacuum. The depth of vacuum must be controlled twice: at the end of the container sealing process and after heat treatment of the product.

The function of vacuum control in technology is performed by special sensors [1, 2]. These are mainly strain gauges, which use a metal membrane with a strain gauge as a sensing element, the deformation of which is converted into electrical signals, which are normalized and digitized into vacuum depth readings. Therefore, vacuum sensors are not autonomous, they require power supply and connection to the display panel. Moreover, the minimum cost of any sensor is much higher than the cost of expensive canned food. The production of canned food is massive and amounts to millions of

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cans. After use, the packaging is thrown away. Therefore, the use of sensors in the packaging is economically impractical and technically impossible.

The closures used in modern glass jar sealing systems are tin lids (Fig. 1). In the center of the lid field are round membranes. They have a small initial deflection in the direction opposite to the load. The membranes can take up two positions: convex or retracted and, depending on this, indicate the presence of a vacuum in the package and thus its tightness. This case is an example of using the energy capabilities of thin plates to detect a vacuum in a package without using sensors.

2. Literature analysis and problem statement

Several methods for calculating thin flexible plates are known today [3, 4, 5]. Depending on the conditions of use of the lids, tinplate of different energy levels is used for their manufacture, which is related to its thickness and hardness [6]. The energy method of calculating thin flat plates is well known and presented in [7, 8], and plates with deflection in [9]. In [10], the general equation of the influence of energy factors on the membrane operation was solved and the equation for calculating the total energy of the membrane was obtained, and the adequacy of the obtained equation with the functioning of real lid membranes was checked.

The analysis of membrane energy levels was initiated in [9] and continued in [10]. Accordingly, the methodology for conducting the experiment to determine additional membrane deflections, including the materials under study, preparation of experimental samples, research procedure, and experimental setup, was used as in [10]. However, no numerical values characterizing the loss of membrane stability and their potential energy were obtained. It is the numerical characteristics of membranes of different thicknesses that make it possible to select them correctly in certain conditions of practical application.

3. Purpose and objectives of the study

The purpose of this work is to create practical opportunities for the use of thin plate energy used to control vacuum in packaging, for possible extension to other areas of application.

Research objectives.

Determine the functioning parameters of membranes of different metal thicknesses.

Perform mathematical modelling of the energy levels of membranes in equilibrium states.

To create a methodology for assessing the energy level of plates and membranes of various shapes, and practical means for its implementation.

4. Materials and methods of research

Materials under study

For a membrane with the following parameters: thickness $\delta = 0.18$ mm, contour fixing radius $R = 12$ mm, and initial center deflection $f_{p.c.} = 0.25$ mm, the design scheme of fixing which is shown in Fig. 1, the additional deflection of its center, a result of loss of stability, will be $f = 0.068$ mm.

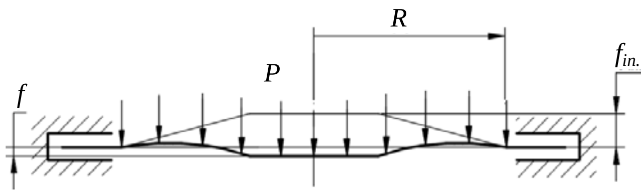

Fig. 1. Design scheme of membrane fixing, P is the load on the membrane

The membrane operates in a controlled loss of stability mode. This mode of operation depends on the energy component of the material, which confirms the feasibility of using this calculation method. In accordance with this method, the total energy of a circular membrane was reflected as the sum of the energies: the unbending stress state, the bending energy, and the work of the external load [9], i.e., the sum of the energies of local effects.

As a result of solving the energy equations of each local effect, we have the equation of the total energy of the membrane system [10]. It can be written in the form:

$$E_n = \frac{\pi}{R^2}(1.4423E\delta^5\zeta^4 + 10.6672D\zeta^2 - 0.3333E\delta^5\zeta P^*), \tag{1}$$

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where $\zeta = \frac{f}{\delta}$ is the dimensionless additional deflection of the membrane center; $D = \frac{E\delta^3}{12(1-\mu^2)}$ is the cylindrical stiffness of the membrane; $P^* = \frac{PR^4}{\delta^4 E}$ is the dimensionless pressure on the membrane; *E is* the modulus of normal elasticity of the membrane material, Pa; μ is the Poisson's ratio of the membrane material.

As a result of substituting *D* into (1) and the value μ = 0.35 for the membrane material, we have:

$$E_n = \frac{\pi E\delta^5}{R^2}(1.4423\zeta^4 + 1.013\zeta^2 - 0.3333\zeta P^*) . \tag{2}$$

For a graphical comparison of the energy levels of the equilibrium positions of different membranes, the dimensionless energy $E_n^* = \frac{E_n R^2}{\pi E\delta^5}$ was introduced. Substituting in E_n^* (2), we obtained the total energy of the membrane in the dimensionless form (3), which characterizes the energy transformations in the membrane body:

$$E_n^* = \frac{E_n R^2}{\pi E\delta^5} 1.4423\zeta^4 + 1.013\zeta^2 - 0.3333 P^*\zeta . \tag{3}$$

Differentiating (3) by *d*ζ and equating it to 0 [14], we derive the equilibrium equation (4):

$$\zeta^3 + 0.3512\zeta - 0.0578P^* = 0. \tag{4}$$

Experimental studies of membrane functioning were carried out under real conditions of using glass container closures for lids of different thicknesses. The parameters $f_{(pf)}$ and *R* were left as in the previous case. The experimental procedure was similar to that described above. The studies showed that the additional deflection of the membrane center in the operating state, which corresponds to the loss of stability, for membranes with parameters δ = 0.16 mm and δ = 0.14 mm was *f* = 0.081 mm and *f* = 0.092 mm, respectively.

For membranes with different tinplate thicknesses, the parameters of the pressure of loss of stability P_1 were determined from the experimental results. First, the dimensionless pressure of loss of stability P_1^* was expressed from (4). Next, P_1^* was determined by substituting the experimental values of *f* for different tinplate thicknesses. Then, the corresponding $P_{(1)}$ for different tinplate thicknesses was determined by expressing it from the dependence for P_1^* as $P_1 = \frac{P_1^*\delta^4 E}{R^4}$. The value of $E = 190\cdot 10^9$ Pa corresponds to mild steel. The results are summarized in Table 1.

Table 1

Functioning parameters of membranes of different thicknesses

δ, m	Parameters			
	f, m	ζ	P_1^*	P_1, Pa
$0.14\cdot10^{-3}$	$0.092\cdot10^{-3}$	0.657	8.48	$0.0301\cdot10^6$
$0.16\cdot10^{-3}$	$0.081\cdot10^{-3}$	0.506	5.3	$0.0323\cdot10^6$
$0.18\cdot10^{-3}$	$0.068\cdot10^{-3}$	0.377	3.41	$0.0334\cdot10^6$

If we graphically represent the functioning of the membrane as a dependence between the pressure (load) *P* on the membrane and the deflection *f* of its center (Fig. 2), then if the pressure reaches the critical value P_1, there will be an abrupt change in the form of the dependence *P*(*f*). At this point, the membrane's working cone loses its stability (wears out). This means that a proper vacuum has been created in the package and it is sealed.

If the pressure continues to increase, this increase will be accompanied by an increase in *f*, which will occur according to a new law different from the initial one. The membrane is unloaded when the pressure reaches a new critical value P_2. In this case, the membrane will again return to the original form of the dependence *P*(*f*). As you can see, since the initial deflection of the membrane is made against the load, it has a negative value. The zero value is the plane of fixing the membrane contour, and the additional deflection is calculated from this plane (Fig. 1).

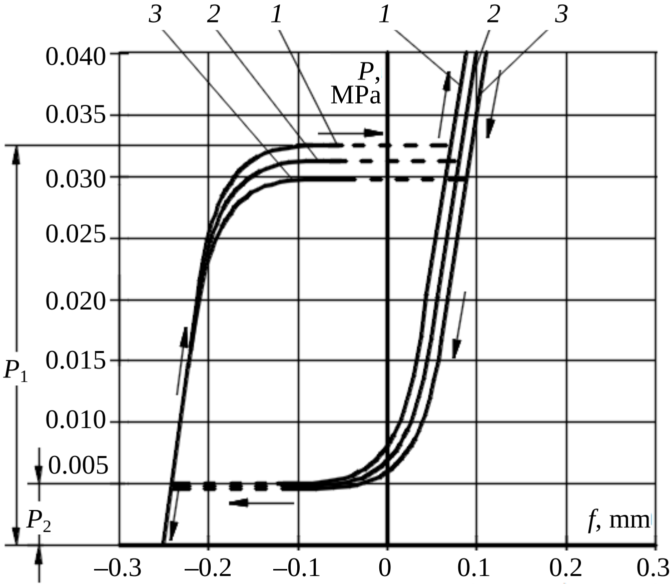

Fig. 2. Relationship between pressure P and deflection f:
1 – δ = 0.18 mm; *2* – δ = 0.16 mm; *3* – δ = 0.14 mm

From Fig. 2 shows that with a decrease in δ, there is a drop in P_1, which is fixed by the membrane. That is, the membrane becomes more sensitive to pressure drop. However, the pressure drop is not significant, based on the specified parameters of membrane functioning. At the same time, to ensure reliable operation of the gate, it is necessary that the membrane fixes a pressure drop of about 0.030 MPa [11].

The research results suggest that the membrane shape change occurs due to the transformation of internal energy under the influence of an external load, as a result of the transition from one equilibrium state to another. In the case of a controlled loss of membrane stability, we are dealing with the potential energy of shape transformation, since the membrane is guaranteed to have only two equilibrium positions, the transition between which is abrupt. Whereas in other cases, an elastic body can have many equilibrium positions depending on the magnitude of the load.

Shape transformation refers to the abrupt transition from one equilibrium state to another. The energy of shape transformation is closely related to the thickness and shape of the membrane cross-section and can be considered a special case of strain energy.

From Fig. 2, we can see that a decrease in the thickness of the tinplate leads to a shift in the right-hand side of the $P(f)$ dependence to the right (Fig. 2), while the left-hand side of this dependence remains in place. In other words, the stroke of the membrane center increases, which means that with a decrease in the membrane thickness, the transformation energy increases, while the pressure of loss of stability decreases.

Modelling

It is proposed to estimate the energy level of membranes of the same shape but different thickness by comparing the energy levels of their equilibrium states. The proposed method is based on the mathematical modelling of the energy levels of the membrane equilibrium state, performed using the energy method from the theory of plates and shells.

The analysis of membrane energy levels is based on the use of (3). Using (3), the dependences of energy levels in the equilibrium positions of membranes made of tin of different thicknesses were plotted graphically. The graphs of dependencies were plotted as functions $E_n^*(\zeta)$ according to the parameters from Table 1 by digital modelling. The zero value of E_n^* of the membrane was taken as the state in which there is no external load on it.

The energy levels of membrane operation corresponding to the state of loss of stability, Fig. 3, reflect the left part of the dependence $P(f)$ from Fig. 2. Based on the fact that in the context of the energy

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component, the critical shape recovery pressure P_2 is not significant due to its small value (about $0.005 \cdot 10^6$ Pa) and the small difference between its values for different tinplate thicknesses, the graphs of energy levels corresponding to the right branch of the $P(f)$ dependence were not shown. In general, the most important thing in this regard is that P_2 does not decrease to 0 or less, since in this case, as can be seen from Fig. 2, the membrane will stop fixing the vacuum in the container.

The graphs (Fig. 3) show that all of the above energy levels have some eigen minimum $E^*_{n\min}$. All three minima reflect the state of loss of stability, and their values increase modulo with decreasing δ. This form of the energy criterion is fully consistent with the well-known Lagrange-Dirichlet theorem, according to which the basic position of a stable equilibrium is the state of the membrane that corresponds to the minimum energy relative to other adjacent states.

5. Analysis of the energy component

If we write E_n from (3) as $E_n = \dfrac{\left|E_n^*\right| \pi E \delta^5}{R^2}$, then taking the value of $E^*_{n\min}$ from the graphs (Fig. 3), we can calculate the minimum energy values for all three membranes. The value $E^*_{n\min}$ is substituted modulo, since the dimensionless energy in the graphs has a negative value. The results are shown in Table 2. Based on the data in Table 2, it can be hypothesised that thinner membranes can accumulate and then return more energy than thicker ones with the same geometric parameters.

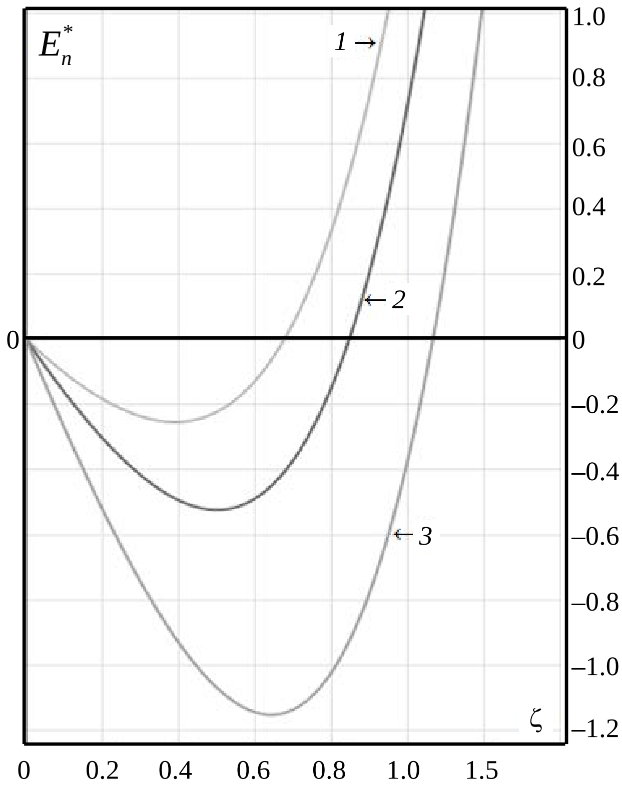

Fig. 3. Energy levels of membranes. Loss pressure of stability P_1:
1 – δ=0.18 mm; *2* – δ=0.16 mm; *3* – δ=0.14 mm

Table 2

Values of the minimum energy of membranes

δ, m	Meaning	
	$\left\|E^*_{n\min}\right\|$	$E_{(n)\,(\min)}$, J
$0.14 \cdot 10^{-3}$	1.161	$0.0822 \cdot 10^{-3}$
$0.16 \cdot 10^{-3}$	0.516	$0.0704 \cdot 10^{-3}$
$0.18 \cdot 10^{-3}$	0.277	$0.0653 \cdot 10^{-3}$

When the membrane is triggered, a sudden transition from one equilibrium state to another occurs. The movement of the membrane during the loss of stability is caused by the external pressure *P*. The transformation energy accumulated in the body of the membrane, which is in a stressed state, is a consequence of the stress concentration in certain areas of the membrane. The maximum stress concentration will be in the peripheral area adjacent to the anchoring contour.

Removing *P* leads to stress dissipation and the membrane instantly returns to its original position (Fig. 1). Physically, this is possible because the rapid transition from one equilibrium state to another is caused by an energy jump. During the energy jump, the transformation energy is released, so the membrane instantly moves in space.

The recovery of shape after loss of stability depends on a favorable geometric shape of the body. Circular membranes have a favorable shape, as the deformation in this case is symmetrical with respect to the center (pole) of the membrane.

Conclusions

The energy component of the membranes of metal closures is analyzed. It is confirmed that the membrane operation can be equated to an autonomous smart energy system that operates without the use of an external energy source, only due to the load drop. The membrane operates in a controlled loss-of-stability mode, restoring its shape due to the transformation energy that can accumulate in the membrane body after deformation and be released by an energy jump after the load is removed.

It is proposed to estimate the energy level of membranes by comparing the energy levels of their equilibrium states. The energy of different membranes in the state of loss of stability is subject to comparison. According to this method, it can be hypothesized that, given the same parameters, the ability to store and release energy by membranes increases with decreasing thickness.

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Ватренко Олександр Віталійович; Oleksandr Vatrenko, ORCID:https://orcid.org/0000-0002-5545-2480

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