# МАШИНОБУДУВАННЯ

## MACHINE BUILDING

**UDC 621.431.74**

**A. Usov**, DSc., Prof.,
**M. Kunitsyn**, PhD, Assoc. Prof.
Odessa Polytechnic National University, 1 Shevchenko Ave., Odesa, Ukraine, 65044; e-mail: usov.anatolij.ua@gmail.com

# MODELING OF QUALITY CHARACTERISTICS AT THE FINISHING OPERATIONS OF WORKPIECES USING VIBRATION RESISTANCE INDICATORS

*А. Усов, М. Куніцин.* **Моделювання якісних характеристик на фінішних операціях деталей за показниками вібростійкості.** Розглядається визначення якісних характеристик оброблюваних поверхонь деталей на фінішних операціях за показниками вібростійкості елементів системи «верстат – пристосування – інструмент – деталь». Поверхня деталі після обробки на фінішних операціях не є гладкою, а завжди має мікроскопічні нерівності, що формують шорсткість та мікродефекти. На операціях, що передують шліфуванню, шорсткість обробленої поверхні впливає на концентрацію напружень, вібраційну активність та утворення теплових дефектів, які під дією термомеханічних явищ, що супроводжують фінішні операції, формують на оброблюваних поверхнях припіки, тріщини та сколи. Процес різання супроводжується інтенсивною вібрацією інструмента, деталі, самого верстата та стружки, що знімається. Поставлена задача та наведений її розв'язок спрямовані на дослідження коливань оброблюваної поверхні деталі під дією сили різання. Результати розрахунків дозволяють оцінити за збігом амплітуд та періоду коливань вібростійкість усієї системи «верстат – пристосування – інструмент – деталь». Запропонований метод визначення зони резонансів силових коливань системи «верстат – пристосування – інструмент – деталь» дає можливість керувати процесом шліфування за умови застосування конкретного інструменту. Це управління здійснюється шляхом вибору швидкостей різання, призначення оптимальних глибин різання або підбору маси оброблюваної деталі. Наведений метод рекомендовано для впровадження у практику оптимальних технологічних процесів шліфування матеріалів, схильних до утворення теплових дефектів. Експериментальна перевірка моделі проведена на деталях зі сталей 102Cr6, X12CrNiTi18-9 та титанових сплавів, що підтверджує точність прогнозування вібростійкості з відхиленнями менше 10 %. Отримані результати дозволили розробити рекомендації щодо вибору режимів і параметрів шліфування, конструкції кріплення та інструменту, що забезпечують стабільність процесу і зменшують утворення теплових і вібраційних дефектів.
*Ключові слова*: фінішні операції, якість поверхні, вібростійкість, амплітуда, період коливань

*A. Usov, M. Kunitsyn.* **Modeling of quality characteristics at the finishing operations of workpieces using vibration resistance indicators.** The paper addresses the determination of the qualitative characteristics of machined workpieces surfaces in finish operations based on the vibration resistance indicators of the "machine – fixture – tool – workpiece" system. After finishing, the surface of the workpieces is never perfectly smooth, but always exhibits microscopic irregularities that form roughness and microdefects. In the operations preceding grinding, surface roughness influences stress concentrations, vibration activity, and the formation of thermal defects that, under the thermomechanical phenomena that accompany finish machining, cause burns, cracks, and chipping of the workpiece. The cutting process generates intense vibrations in the tool, the workpiece, the machine itself, and the chip removed. The stated problem and its proposed solution focus on studying the oscillations of the workpiece surface under the action of cutting forces. Calculation results make it possible to evaluate the vibration resistance of the entire "machine – fixture – tool –workpiece" system by matching oscillation amplitudes and periods. The proposed method for identifying the resonance zones of the system's force oscillations enables control of the grinding process when using a specific tool. This control is achieved by selecting cutting speeds, determining optimal cut depths, or adjusting the workpiece mass. The method is recommended for implementation in the practice of optimizing grinding processes for materials prone to thermal defects. Experimental validation of the model was carried out on workpiecess made of 102Cr6 and X12CrNiTi18-9 steels and titanium alloys, confirming prediction accuracy within 10% deviation. The results obtained have allowed the development of recommendations for choosing grinding regimes and parameters, as well as fixture and tool design, to ensure process stability and reduce the occurrence of thermal and vibration-induced defects.
*Keywords*: finishing operations, surface quality, vibration resistance, amplitude, oscillation period

## Introduction

Unlike an ideal surface, the surface of a workpiece after finishing operations is not smooth, but always has microscopic irregularities that form roughness. Despite the rather small size of the irregularities that form roughness, they have a significant impact on various operational properties of workpieces [1, 2]. In the operations preceding grinding, the roughness of the machined surface affects the concentration of stresses, vibration activity, and the formation of thermal defects, which, under the influence of thermomechanical phenomena accompanying finishing operations, form cauterization,

**DOI: 10.15276/opu.1.71.2025.01**

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

cracks, and chips on the machined surfaces [3 – 6]. Numerous theoretical and experimental studies have been devoted to the study of this process [7 – 13]: the effect of grinding wheel wear, the creation of macro- and microrelief of the ground surface, fluctuations in thermomechanical parameters in the contact zone of the grinding wheel with the treated surface, and vibration of the “machine – fixture – tool – workpiece” (MFTW) system.

**Analysis of recent research and publications**

Finishing operations are a critical stage in the manufacture of workpieces that determines their performance and reliability. The quality of the machined surface, including roughness, defects, residual stresses, and structural changes in the surface layer, is directly dependent on the process parameters. The dynamic processes that occur in the elastic system of the machine, tool, and workpiece, in particular vibrations, play an significant role in the formation of these characteristics [8, 11, 14, 15]. Understanding and modeling the effect of vibrations on surface quality and tool condition is an urgent task in order to improve efficiency and predict the results of finishing.

Research confirms that the dynamic interaction between the tool and the workpiece is an integral part of many finishing operations, such as grinding [14, 15], friction machining [8], and vibration machining [5]. In the grinding process with a diamond metal pencil, the relative vibrations of the wheel and the dressing tool affect the roughness of the surface of the ground [10]. During intermittent grinding, where the wheelwork surface consists of alternating protrusions and depressions, there is a periodic change in the stiffness of the machine tool dynamic system [15], which can lead to oscillations, in particular, parametric resonance [11, 14, 15]. Similarly, during friction machining with a tool with transverse grooves (intermittent working part), intermittent contact is created, causing shock loads and dynamic processes [8].

Modeling the elastic system of a machine tool for different types of machining allows us to study these dynamic processes. For the frictional machining of flat surfaces, a calculation scheme of the elastic machine tool system was developed in the form of a multi-mass model (three-mass model), where the masses of the nodes are connected by elastic and damping links, and the interaction between the tool and the workpiece is modeled by contact stiffness and energy damping [8]. Such a model allows obtaining the amplitude-frequency characteristic of the machining process and determining the system parameters at which resonance is possible [8, 15]. The study of the parametric stability of the elastic system of a surface grinding machine was also carried out using mathematical models that allow us to identify the conditions for the occurrence of parametric resonances and areas of instability [15]. An increase in damping has been shown to lead to a narrowing of the boundaries of the regions of unstable operation of the elastic machine [15].

The strengths of these studies are the development of mathematical models capable of describing complex dynamic interactions in the MFTW-system and predicting the conditions for resonance and instability [8, 15]. Weaknesses may be associated with simplifications used in model development (e.g., a finite number of masses, linear or piecewise-constant characteristics of the connections), which may limit the accuracy of prediction in real conditions.

Vibrations and parametric resonance have a direct and often negative impact on the results of finishing. During intermittent grinding, large-amplitude vibrations occurring in an elastic system lead to a deterioration in the geometric characteristics of the surface layer quality of the workpiece and an increase in the dimensional wear of the intermittent grinding wheel [14]. Parametric resonance can be accompanied by catastrophic wear of the abrasive tool and deterioration of the geometric and physical and mechanical characteristics of the surface layer quality [11]. Studies of the dynamics of discontinuous grinding confirm this relationship [14]. Stable operation of the elastic system, which ensures burn-free machining and efficient material removal, can be achieved when working with discontinuous circles with a large number of depressions and the correct choice of cutting mode parameters [14].

The condition of the tool’s working surface is also closely related to dynamic processes. The grinding wheel dressing conditions affect the condition of its working surface and the level of chipping of the grain tips [10]. The result of the grinding process depends on the location of the grains and the dressing conditions [10]. Editing conditions affect the stability of the grinding wheel and the self-sharpening process [10]. When grinding polycrystalline superhard materials (PCSM) with diamond wheels, the self-sharpening process occurs periodically, which manifests itself in the frequency of changes in the physical and technological parameters of the processing, including the tangential cutting force [16]. This adaptation of the grinding system makes it possible to determine the effective component of the tangential cutting force, which directly determines the removal process [16]. The intensity of self-sharpening and grain wear depends on the height of their protrusion above the level of bonding and contact interaction [16].

In vibromachining, the impact contact of an abrasive granule with the surface of a workpiece leads to the formation of characteristic marks and the formation of surface relief [5]. The depth of the mark depends on the speed of movement, force, and penetration frequency [5]. The alternation of abrasive grains (friction, plastic pressing, cutting) is related to the nature of the movement of the granules [5].

Another important quality characteristic is residual stresses. Although sources mainly consider residual stresses in coatings obtained by gas-thermal spraying [9], they recognize that the magnitude and distribution of these stresses depend on many factors, including process parameters [9]. A mathematical model has been developed to determine residual stresses in the coating by measuring the deflection of the sample, allowing one to predict their level [9]. Although this work does not directly relate residual stresses to machine or sputtering system vibration, it emphasizes the importance of modeling these stresses to predict performance [9].

Sources also mention stochastic defects that form in the surface layer of metal products with a heterogeneous structure during machining [12]. The causes of structural changes and crack formation are investigated depending on the probability distribution of defects and heat flux. Thermomechanical processes of the machined surface are modeled based on equations of thermoelasticity [12].

The analysis of the presented sources reveals several gaps and areas for further research.

– Although models of the dynamic system of a machine tool have been developed and allow resonance studies [8, 15], the direct, quantitative relationship between specific indicators of vibration resistance (e.g. damping level, distance from resonant frequencies) and a wide range of surface qualities (roughness, residual stresses, microstructure, defects) is not always clearly represented in the form of generalized models.

– Modeling residual stresses in coatings [9] describes in detail the calculations based on deflection, but does not study the influence of dynamic factors or vibration of the equipment to generate these stresses.

– Studies of grinding roughness have focused mainly on dressing conditions and abrasive grain geometry [10], paying less attention to the direct effect of machine vibration during the grinding process itself on roughness.

– Although the dynamic processes in intermittent grinding and friction machining with intermittent tools have been studied [8, 11, 13, 14, 15], the generalization of these models for other finishing operations where vibrations are also present (e.g., turning, milling, honing) is not observed in these sources.

– There is a gap between models that describe the macroscopic dynamic behavior of the MFTW-system [8], [15] and models that describe microscopic processes of tool-material interaction and surface layer formation (self-sharpening [16], trace formation [5], nanocrystalline structure formation [8]). Integrating these levels of modeling is a challenging task.

In contrast to studies focused on specific processes (grinding [10, 11, 13 – 17], spraying [9], friction machining [8], vibration machining [5]), our study seeks to identify general patterns of impact of vibration on quality that apply to different technologies. This may include:

– Development of universal vibration resistance indicators relevant for quality assessment regardless of process specifics.

– Create models that predict changes in a set of quality characteristics (not just one, such as roughness or residual stress), depending on the dynamic state of the system.

– Investigation of the relationship between machine tool macrodynamics and microprocesses of surface layer formation under the influence of vibrations.

Thus, our study, based on existing work on the dynamics of machine tool systems and modeling of individual quality characteristics, seeks to create a more comprehensive and generalized approach that will allow prediction and control of the quality of finishing based on monitoring and controlling the vibrodynamic state of the technological system.

The reviewed literature demonstrates a significant amount of research on certain aspects of finishing, dynamic processes in machine tools, and the formation of surface quality characteristics. A clear link has been found between vibrations, especially those that occur during intermittent contact or as a result of parametric resonance, and deterioration in surface quality, as well as increased tool wear

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

[11, 14], 15]. Mathematical models have been developed to analyze the dynamics of machine tool systems [8, 15] and to model specific quality indicators, such as roughness (associated with dressing) [10] and residual stresses (for spraying) [9].

However, the existing literature has some gaps, especially in terms of comprehensive and generalized modeling of the impact of vibration resistance on a wide range of surface quality characteristics for different finishing operations. Your future research is important because it aims to overcome these limitations by offering a more universal approach to understanding and predicting the relationship between the vibrodynamic state of a process system and the final quality of machined workpieces during the finishing stages. The results of such a study can be widely used in practice to optimize technological processes, improve product quality stability, and reduce production costs.

**The purpose and objectives of the research**

The aim of the study is to develop and experimentally substantiate a method for mathematical modeling of the quality characteristics of the surface of workpieces during finishing operations based on the vibration resistance of the elements of the MFTW-system, which allows to identify zones of resonant vibrations and control technological parameters (cutting speeds and depths, workpiece weight) to prevent resonance and ensure optimal quality of the machined surface of materials prone to thermal defects.

**Materials and methods of the research**

It is known [11, 18] that the grinding process has a wave character: under the action of cutting forces on a material with known compliance, shock waves are generated. The cutting operation is accompanied by intense vibration of the tool, the workpiece, the machine tool itself, and the removed chips within the MOTS volume [19, 20]. Let us consider the vibrations of a workpiece (dimensions $L_0$, $B_0$, $H_0$), clamped to the machine table. We adopt the following assumptions.

1. Plastic state of the removed layer. In the grinding process, for $0.01 \le t \le 0.1$, at the onset of cutting the material in the removed layer behaves plastically. Therefore [21], [22]:

$$k_\rho = \begin{cases} 1 & \text{over the segment } \Delta L_0, \\ \dfrac{P_z}{B_0 H_0 E} & \text{over the remaining length.} \end{cases}$$

2. Deformation-dependent roughness parameter. The parameter $y$ is taken as a function of the workpiece deformation $\Delta L_0$ [23].

3. Material damping coefficient. The coefficient $C_\delta$ is determined for the chosen workpiece material [24].

4. Distributed cutting force. Vibrations are considered throughout the workpiece: At the time of grain engagement, the cutting force contributed by all active grains in the contact zone acts uniformly over the surface of the workpiece along the cutting axis [25].

5. Duct protection system clamping. The effect of the workpiece clamping to the machine table is modeled as damping with known compliance $\delta_t = \delta_z + \delta_y$ [26].

Let's write the equation of vibrations of the machined surface of the workpiece under the action of the cutting force $P(y,z,t)$ as a function of $u(t,y,z)$, which satisfies equation [18]:

$$\frac{\partial^2 u}{\partial t^2} = a^2\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{1}{\rho} P(y,z,t), \tag{1}$$

with the initial conditions:

$$u\ (z,y,0) = u_0,\ \frac{\partial u}{\partial t}(z,y,t_0) = P_{zz}\sin(\omega t_0 + \varphi_0), \tag{2}$$

and boundary conditions:

$$u(0,y,0) = 0,\ u(L,y,0) = 0,\ u\ (z,0,0) = \delta_z,\ u\ (B,y,0) = \delta_y. \tag{3}$$

The solution to the problem using the methods of integral transformations [27], [28] is as follows:

$$u(t,y,z)=\frac{2hl^2\delta}{\pi^2 c(l-c)}e^{-mt}\sum_{k=1}^{\infty}\frac{1}{k^2}\sin\frac{k\pi z}{l}\sin\frac{k\pi y}{l}\left(\cos\omega_k t+\frac{m}{q_k}\sin\omega_k t\right), \quad (4)$$

where:

$$\omega_k=\sqrt{\left(\frac{k\pi a}{l}\right)^2-m^2}.$$

The frequency of the forced oscillations during the period of time of the $P_z$ values may coincide with the natural frequency of the workpiece. This coincidence is dangerous for machining, as a force resonance may occur in the tool- workpiece system, which can lead to tool destruction or deterioration of the machined surface quality.

The results of the calculations allow us to estimate the vibration resistance of the entire MFTW-system by the coincidence of the amplitudes and the period of oscillations. Thus, in our approach to determine the law of oscillation of a part in the direction of the *OZ* axis [27], we will:

$$Z=a_0\cdot e^{-\psi\cdot T}\cdot\sin(\sqrt{\omega^2-\psi^2}\cdot T+\varphi), \quad (5)$$

where: $a_0$ is the initial amplitude of the workpiece's vibration after the impact load, in meters; $\psi$ is the damping coefficient of the MFTW-system, accounting for the compliance of the clamping, in, s$^{-1}$; $T$ is the elapsed time from the onset of the vibration process, in seconds; $\varphi$ is the initial phase of the vibration (phase shift), in radians; $\omega$ is the natural (angular) frequency of the workpiece's vibration, in rad/s, calculated by the formula:

$$\omega=\sqrt{\frac{C_\delta}{L_0 H_0 B_0 \rho}}, \quad (6)$$

where: $C_\delta$ – is the stiffness (elasticity) coefficient of the MFTW-system, in N/m, calculated by the formula:

$$C_\delta=\frac{\{L\}EH_0B_0}{L_0^2},$$

where: $L_0$, $H_0$, and $B_0$ – are the mean geometric dimensions of the workpiece–length, height, and width, respectively, in meters; $\rho$ – is the density of the workpiece material, in kilograms per cubic meter (kg/m³):

$$\psi=\frac{k_\rho}{2m}=\frac{\{m\}\cdot k_\rho}{2m\cdot\{T\}}=\frac{P_z\cdot\{m\}}{2H_0^2\cdot B_0^2\cdot L_0\cdot\rho\cdot E\cdot\{T\}}, \quad (7)$$

where: $\psi$ is the damping coefficient of the MFTW-system, accounting for clamping compliance, in s$^{-1}$; $k_\rho$ is the damping constant (damping-force coefficient) of the clamping system, with units N·s/m; $m$ is the mass of the workpiece, in kg; $\{m\}$ denotes the unit of length (meter), shown in braces to ensure dimensional consistency during transformations; $\{T\}$ denotes the unit of time (second), shown in braces to unify dimensions in the expression; $P_z$ is the force pressing the workpiece against the machine table – determined by the cutting parameters and the shear angle – in *N*; and $E$ is the Young's modulus (elastic modulus) of the workpiece material, in Pa.

Thus, the transition from the original expression in (7) to the expanded form explicitly takes into account the dependence of damping on the clamping force, workpiece geometry, material, and units of measurement.

Determine the workpiece clamping force $P_y$ from $P_z$ and the chip cutting angle by replacing the curved section with a straight line. Given the small value of $t$ compared to the grinding wheel radius $R$, the error will be less than a fraction of a percent.

According to Fig. 1, there will be:

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

$$P_y = P_z \frac{\sqrt{2Rt - t^2}}{R - t}, \tag{8}$$

$$\Delta L_0 = \sqrt{P_z^2 + P_y^2}\,\frac{L_0}{EH_0B_0} = \frac{R}{R-t}\frac{L_0}{EH_0B_0}P_z. \tag{9}$$

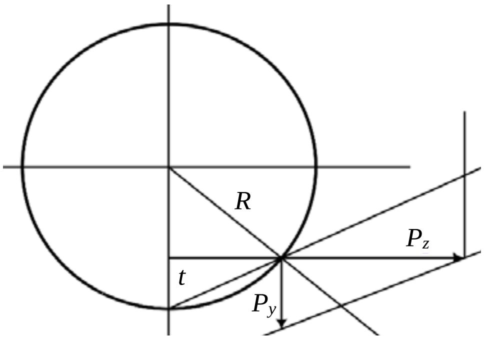



**Fig. 1.** Diagram of cutting forces during grinding

Then the amplitude of the oscillations is determined by the formula [22]:

$$A_0 = \sqrt{\frac{R^2}{(R-t)^2}\frac{L_0^2 \cdot P_z^2}{E^2 \cdot H_0^2 \cdot B_0^2} + \frac{(V_c + V_f)^2 \cdot L_0^3 \cdot \rho}{E \cdot \{L\}}}.$$

The initial conditions are obvious: $z(0) = \Delta L_0$. From here:

$$\begin{cases} A_0 \sin y = \dfrac{R}{R-t}\dfrac{L_0}{EH_0B_0}P_z; \\ y = \arcsin\sqrt{\dfrac{\Delta L_0^2}{\Delta L_0^2 + \dfrac{(V_c+V_f)^2 \cdot L_0^3 \cdot \rho}{E \cdot \{L\}}}}. \end{cases} \tag{10}$$

The final amplitude will look like this:

$$Z = A_0 \cdot e^{-\psi \cdot T} \cdot \sin\left(\sqrt{\omega^2 - \psi^2} \cdot T + \arcsin\sqrt{\frac{\Delta L_0^2}{\Delta L_0^2 + \dfrac{(V_c+V_f)^2 \cdot L_0^3 \cdot \rho}{E \cdot \{L\}}}}\right). \tag{11}$$

At the same time, there are correlations:

$$m = L_0 H_0 B_0,\ \omega = \sqrt{\frac{E\{L\}}{L_0^3 \rho}},$$

$$\omega^2 - \psi^2 = \frac{E\{L\}}{L_0^3 \rho} - \frac{P_z^2 \cdot \{m\}^2}{4H_0^4 \cdot B_0^4 \cdot L_0^2 \cdot \rho^2 \cdot E^2 \cdot \{T\}^2},$$

$$\frac{\omega^2 - \psi^2}{\omega^2} = 1 - \frac{P_z^2 \cdot \{m\}^2 \cdot L_0}{4H_0^4 \cdot B_0^4 \cdot \rho \cdot E^3 \cdot \{T\}^2 \cdot \{L\}}.$$

The amplitude $Z$ is determined by the measuring complex [9]. The experimental results were obtained in the form of oscillograms of the oscillatory process, which are changes in the values of cutting forces over time. The cutting force $P_z$ determines the law of oscillation of the workpiece and is taken as the total for all cutting grains at the moment of plunging. The size of $Z$ depends on the time $T$. The oscilloscope records these values of the time the grinding wheel passes over the entire surface to be machined. To determine the resonance characteristics, consider the maximum amplitude values.

In the absence of resonance, the maximum amplitude value should be expected after the elastic compression of the workpiece at the moment of insertion, that is, after the impact load and the first elastic compression wave occur. The experiment makes it possible to determine the greatest force of the impact load $P_z$ and its duration is the impact phase. Theoretically, this time can be determined by equating the first derivative of $Z$ or $P_z$ to zero, and practically from the graph taken from the oscilloscope. Thus, to solve the problems of optimizing grinding processes formalized in the previous sections, it is necessary to determine the acting cutting forces and the vibration field of the MFTW-system. These are two successive tasks.

The first is to determine the power characteristics of the grinding process:

$$P_z = \frac{v}{\delta k_1 c} \cdot e^{-k_2 cT} \cdot \sin k_1 cT \to \max.$$

$$\begin{cases} k_1 = \sqrt{\dfrac{1}{E \cdot F \cdot \varsigma \cdot L \cdot \delta} - \dfrac{1}{(2EF\delta)^2}}, \quad k_2 = \dfrac{1}{2EF\delta}, \quad \dfrac{cT_{\max}}{L} = \pi\sqrt{\varsigma}, \quad T_{\max} = \dfrac{L}{V_c}; \\ L = \sqrt{2Rt - t^2}, \quad h = l\dfrac{V_f}{V_c}\dfrac{R-t}{R}, \quad k_1 \cdot c \cdot T_{\max\ \text{nep}} = n \cdot 2\pi (n = 1, 2, 3\ldots); \\ [a] = a, \quad [b] = b, \quad [l] = l, \quad [t] = t, \quad [\tau_n] = \tau_n, \quad P_{-z} = (a + 2h) \cdot \Delta L \cdot [\tau_n]. \end{cases}$$

Given a given range of changes in $V_c$ and $V_f$ for a particular machine tool, this task is solved according to the scheme of Fig. 2.

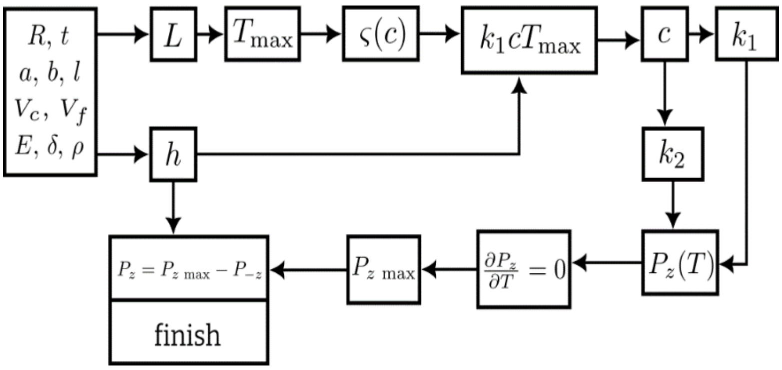



**Fig. 2.** Calculation scheme to determine the maximum cutting impact forces

**Research results**

The solution algorithm based on the scheme shown below and the determination of the maximum cutting forces according to the scheme in Fig. 2 are described in [29]. Here we present several examples of grinding workpieces made from materials prone to thermal defects (102Cr6 (1.2067); 90CrSi / 90CrSi5 / 150Cr14 (DIN WNr), and X12CrNiTi18-9 (EN 1.4541) steels; Ti 6242 (Ti-6Al-2Sn-4Zr-2Mo) and Ti 3Al-2.5V (Grade 9) or Ti 6Al-4V (Grade 5) titanium alloys). Consider the use of a diamond grinding wheel with: Radius $R = 6.25$ cm; cutting depths $a = 0.004$ cm, $b = 0.006$ cm; engagement length $l = 0.012$ cm; for the steel specimens we used: Young's modulus $E = 2 \cdot 10^7\ \text{N/cm}^2$, effective weight per unit volume $\rho \cdot g = 0.008\,\text{kg/cm}^3$, fixture compliance $\delta = 0.015\,\text{cm/N}$; specimen dimensions were: $H_0 = 1.5$ cm, $B_0 = 2.0$ cm, $L_0 = 5.5$ cm.

Cutting speed:

$$V_c = \begin{Bmatrix} 2618\ \ \text{cm/s at}\,4000\ \text{rpm} \\ 2618\ \ \text{cm/s at}\,4000\ \text{rpm} \\ 2618\ \ \text{cm/s at}\,4000\ \text{rpm} \end{Bmatrix}.$$

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

Feed velocity ranges from 0 to 1 cm/s. Depth of cut: $t = \{0.0005\,\text{cm}; 0.001\,\text{cm}; 0.0015\,\text{cm}\}$. For a cutting speed $V_c = 2618\ \text{cm}/s$ and $t = 0.0005\,\text{cm}$, according to the computational scheme (Fig. 2), we obtain:

$$T_{\text{пер}} = 17\cdot 10^{-5}\, s,\ c = 4319.526846\ \text{cm}/s,$$

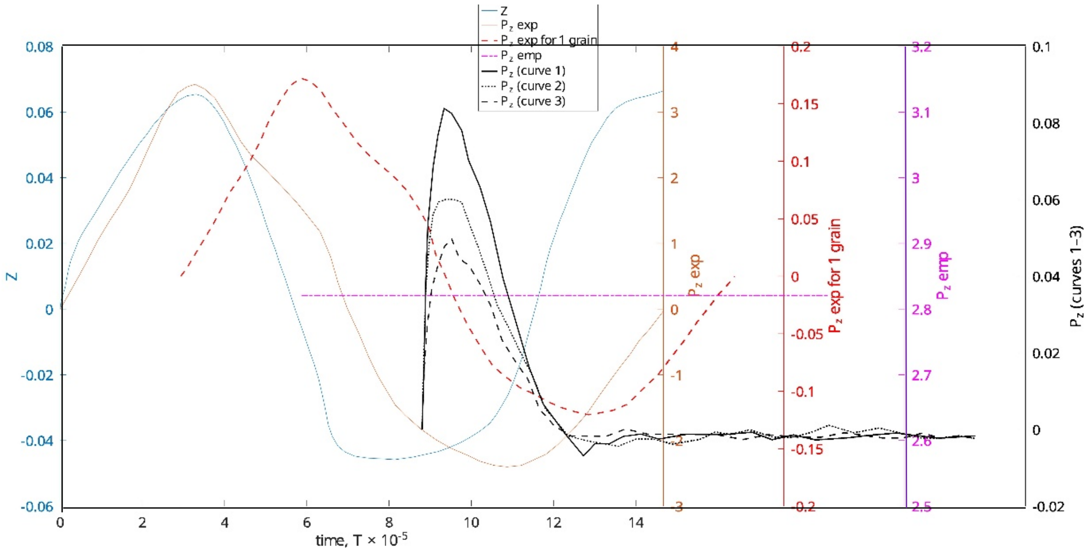
$$P_z(T) = 0.4724\cdot e^{-392713T}\sin 35960T,$$

$$P_{z\,\max}(T = 0.2993\cdot 10^{-5}\, s) = 0.1875\ \text{N}.$$

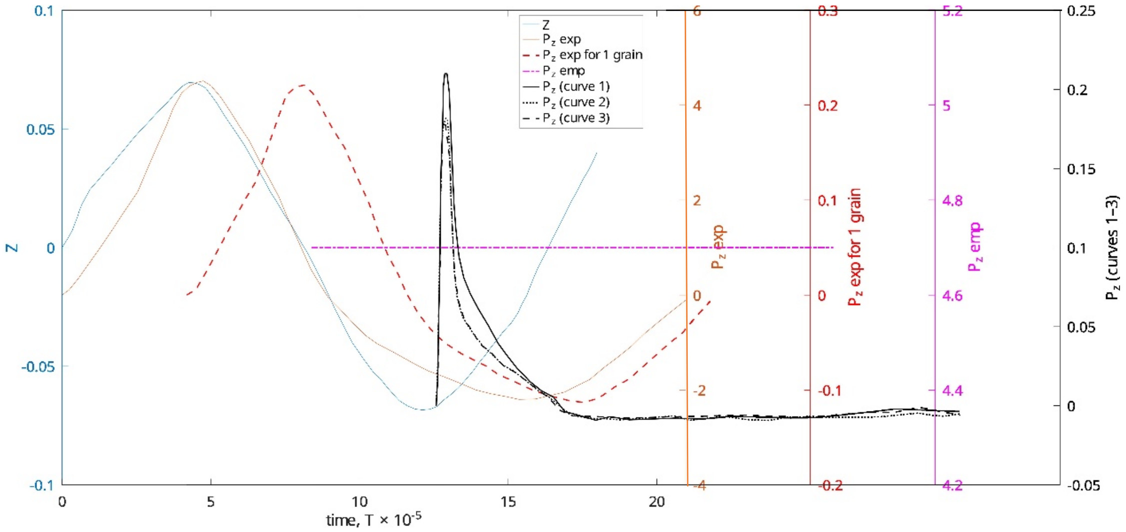
For titanium alloys Ti 6242 (Ti-6Al-2Sn-4Zr-2Mo) and Ti 3Al-2.5V (Grade 9) or Ti 6Al-4V (Grade 5):

$$\begin{cases} P_z(T) = 0.4570\cdot e^{-394707\cdot T}\sin 26180T; \\ P_z(T) = 0.4683\cdot e^{-333992\cdot T}\sin 29133T; \\ P_z(T) = 0.4707\cdot e^{-301007\cdot T}\sin 32191T. \end{cases}$$

The graph of the dependencies $P_z(T)$ are shown in Fig. 3.




*a*




*b*

**Fig. 3.** Time dependences of $P_{z\ \text{exp for a single grain}}$, $P_{z\ \text{exp}}$, $P_{z\ \text{emp}}$, $P_z$ and $Z$: *a* – for titanium alloys; *b* – for steels

Obviously, $P_z$ (excluding $P_{-z}$) generates an impact impulse that induces oscillations in the MFTW-system. However, for this process one must use the total cutting force from all active abrasive grains on the grinding wheel surface at the moment of engagement. The condition $P_z - P_{-z} > 0$ must be satisfied, otherwise no cutting will occur.

For the case of $t = 0.001$ cm:

$$c = 3627 \text{ cm/s}; \; P_{z\,\max} = 0.194\text{N}; \text{ and } P_z(T) = 0.4752 \cdot e^{-329771 \cdot T} \sin 36744T.$$

For the case of $t = 0.0015$ cm:

$$c = 3020 \text{ cm/s}; \; P_{z\,\max} = 0.214\text{N}; \text{ and } P_z(T) = 0.4807 \cdot e^{-299274 \cdot T} \sin 36319T.$$

The graphs of dependencies are shown in Fig. 3. These figures show the results of experimental measurements of $P_z$ values. The values of the maximum theoretical and experimental $P_z$ are given in Table 1.

**Table 1**

Values of maximum theoretical and experimental values of $P_z$

| $V$ m/s | 26.19 | | 17.74 | | 9.85 | |
|---|---|---|---|---|---|---|
| $t$ µm | $P_{z\text{ theor}}$ | $P_{z\text{ exp}}$ | $P_{z\text{ theor}}$ | $P_{z\text{ exp}}$ | $P_{z\text{ theor}}$ | $P_{z\text{ exp}}$ |
| 5 | $0.188N_{01}$ | 0.225 | $0.195N_{01}$ | 0.235 | $0.200N_{01}$ | 0.240 |
| 10 | $0.194N_{02}$ | 0.233 | $0.205N_{02}$ | 0.246 | $0.215N_{02}$ | 0.260 |
| 15 | $0.214N_{03}$ | 0.257 | $0.225N_{03}$ | 0.270 | $0.240N_{03}$ | 0.289 |

As shown in Table 1 (where $P_z$ is given in newtons and $N_{ij}$ denotes the number of cutting grains in the tool-workpiece contact zone during cutting), the theoretical values of $P_z$ exceed the measured ones by 15…20 %. This discrepancy is to be expected, since the machine tool's compliance was not taken into account in the theoretical calculation of $P_{z\text{ theor}}$. In practice, this effect can always be incorporated. In essence, the problem has been fully resolved with respect to cutting regimes, temperature, and surface roughness – the latter being simply verified by analytical formulas. Should the temperature or the micro-roughness height exceed the prescribed limits, the peripheral speed is reduced.

Table 2 presents the computed values of cutting-zone temperature and the height of surface micro-roughness. The table also lists the permissible temperature limits in the cutting zone.

**Table 2**

The computed values of cutting-zone temperature and the height of surface micro-roughness

| $V_c + V_f$ cm/s | $t$ µm | Steel 102Cr6 (1.2067) | | | Ti-6Al-4V (Titanium Grade 5) | | | Steel X12CrNiTi18-9 (EN 1.4541) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\theta_{calc}$C | $[\theta]_{calc}$C | $y_{0\text{ calc}} \cdot 10^{10}$ cm | $\theta_{calc}$C | $[\theta]_{calc}$C | $y_{0\text{ calc}} \cdot 10^{10}$ cm | $\theta_{calc}$C | $[\theta]_{calc}$C | $y_{0\text{ calc}} \cdot 10^{10}$ cm |
| 2619 | 0.0005 | 710 | 700 | 0.190 | 930 | 880 | 0.27 | 800 | 770 | 0.18 |
| | 0.0010 | 730 | 700 | 0.230 | 970 | 880 | 0.31 | 810 | 770 | 0.19 |
| | 0.0015 | 740 | 700 | 0.263 | 985 | 880 | 0.36 | 830 | 770 | 0.23 |
| 1774 | 0.0005 | 650 | 700 | 0.170 | 900 | 880 | 0.25 | 790 | 770 | 0.17 |
| | 0.0010 | 670 | 700 | 0.190 | 920 | 880 | 0.27 | 770 | 770 | 0.18 |
| | 0.0015 | 680 | 700 | 0.210 | 930 | 880 | 0.29 | 750 | 770 | 0.19 |
| 985 | 0.0005 | 640 | 700 | 0.160 | 880 | 880 | 0.23 | 730 | 770 | 0.16 |
| | 0.0010 | 650 | 700 | 0.180 | 850 | 880 | 0.25 | 700 | 770 | 0.17 |
| | 0.0015 | 665 | 700 | 0.200 | 820 | 880 | 0.26 | 690 | 770 | 0.18 |

Naturally, because the influence of the cooling/lubrication system was not included, the simulated temperature field is overestimated. Experience shows that this overestimation is on the order of 10 %, which is fully compensated for when cooling/lubrication system is applied. It should be noted that the permissible temperatures for titanium alloys in Table 2 are based on the range required for the

 ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

allotropic transformation of titanium from a hexagonal close-packed (HCP) to a body-centred cubic (BCC) structure (880…900°C); on the grinding surface this transformation can begin as low as 450°C. Finally, the variable parameters-circumferential speed ($V_c$), feed speed ($V_f$) and depth of cut ($t$)-are adjusted to determine the optimal grinding process.

Table 2 shows that the roughness requirements in the cutting-grain contact zone are met automatically. To verify the surface roughness produced between two adjacent cutting grains, one substitutes $\tau=[\tau_n]$, $x=0.5l$, $q_x=[\sigma]$ into the roughness formula. In this case, $t-y_{\max}$ defines the height of the micro-roughness. The lines of influence (i.e., of maximum stress) are second-order curves; therefore, in practice, material cracking during cutting will proceed along the tangents to the planes of these influence lines, since the detachment speed is very high and the stress concentration at the crack tip reaches its critical value. The intersection of two opposite planes thus determines the micro-roughness height. For 102Cr6 (1.2067) steel, the calculations yield an average micro-roughness height of $y_{0\,\text{calc}}=\{0.19\cdot10^{-10}-0.3\cdot10^{-5}\}$ cm for the chosen cutting depths. Next, we consider the vibration-resistance index (VRI). The amplitude of oscillation $Z$ of the workpiece can be determined by solving the corresponding system of equations. The calculation scheme is shown in Fig. 4.

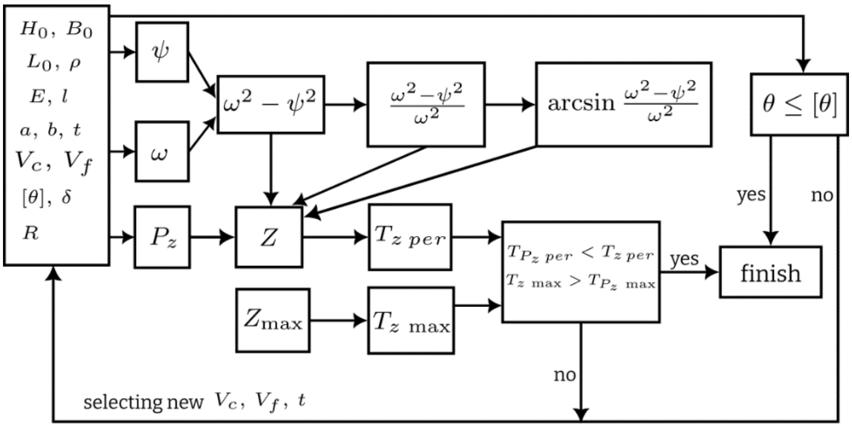



**Fig. 4.** Calculation scheme for optimizing the grinding process using the vibration resistance criterion

The program to implement the selection of the optimal grinding technological process is presented in [28]. The values $L_0$, $B_0$, and $H_0$ denote the average dimensions of the geometric characteristics of the workpiece; these can be refined based on the specified mass of the workpiece. A large number of experiments were carried out on the measurement rig to validate the calculated data using rectangular-bar specimens of various steels with dimensions $L_0=5.5$ cm $H_0=1.5$ cm, and $B_0=2$ cm. The empirical relationships for different cutting depths are given in [30, 31]:

$$\begin{cases} Z(T)=0.0682e^{-6.19\cdot10^{-8}P_zT}\sin\left(38374T+\dfrac{13.4\cdot10^{-6}P_z}{\{P_z\}}\right), & t=0.0005\,\text{cm};\\ Z(T)=0.0682e^{-6.187\cdot10^{-8}P_zT}\sin\left(38374T+\dfrac{13.43\cdot10^{-6}P_z}{\{P_z\}}\right), & t=0.0001\,\text{cm};\\ Z(T)=0.0682e^{-6.187\cdot10^{-8}P_zT}\sin\left(38374T+\dfrac{13.44\cdot10^{-6}P_z}{\{P_z\}}\right), & t=0.0015\,\text{cm}. \end{cases}$$

The periodicity of the functions $Z(T)$ is essentially identical and equals $T_{z\,\text{theor}}=16.37\cdot10^{-5}$ s. Given the small magnitude of $P_z$, the theoretical peak values of $Z$ are very close to one another, each amounting to 0.0682 cm. Fig. 2 shows the plots of $Z$ versus time. As can be seen, the oscillation periods of the cutting forces and of the MFTW-system differ, yet are close in value, so the possibility of resonance cannot be ruled out.

The proposed method to identify the resonance zones of the force oscillations in the MFTW-system allows the process engineer to control the grinding operation for a given tool. This control can be exercised by selecting appropriate cutting speeds, prescribing optimal depths of cut, or by varying (or by adding) the mass of the workpiece via connection of technological masses. Results shows that shifting the phase and the maximum amplitudes of the oscillations in both the cutting force and the MFTW-system vibration by only 10…15 % completely suppresses resonance. However, as Fig. 2 indicates, this strategy does not yield high accuracy in determining the values of $P_z$. Moreover, it produces a constant – and significantly overestimated – cutting force, which does not reflect the real grinding process.

The Saint-Venant-Boussinesq approach [18, 32], by contrast, provides a sufficiently close approximation to experimental data, with deviations rarely exceeding 7…9 % of the measured values [16, 17]. This allows us to recommend this method for implementation in the formalization of optimal technological grinding processes for materials prone to thermal defects.

**Conclusions**

The proposed method for identifying the resonance zones of force oscillations in the MFTW system makes it possible to control the grinding process when using a specific tool by selecting appropriate cutting speeds, prescribing optimal depths of cut, or varying (or adjusting) the mass of the workpiece. Theoretical and experimental investigations have shown that shifting the phase and the maximum amplitudes of both the cutting-force oscillations and the MFTW-system vibrations by only 10…15 % completely eliminates the occurrence of resonance. However, while this strategy effectively suppresses resonance, it does not yield high accuracy in determining cutting-force values and tends to overestimate them, which is not reflective of the real grinding process. By contrast, application of the Saint-Venant–Boussinesq approach provides close agreement with experimental data, with deviations rarely exceeding 7…9 %, and is thus recommended for formalizing optimal technological grinding processes for materials prone to thermal defects.

Extensive validation – including tests on specimens of 102Cr6 and X12CrNiTi18-9 steels and various titanium alloys – confirmed the model's prediction accuracy within 10 % deviation, demonstrating its robustness across diverse material properties and geometries. In particular, measurements of the maximum cutting forces ($F_{max}$) showed that theoretical values exceed the experimentally measured ones by 15…20 % when machine-tool compliance is not accounted for; incorporation of the finite stiffness of the machine–fixture system in the model reduces this discrepancy to within experimental error bounds. Furthermore, computed values of cutting-zone temperature and surface micro-roughness for steel (e.g., 102Cr6) and titanium-grade workpieces proved sufficient to meet prescribed limits (e.g., micro-roughness heights on the order of 0.16…0.27 μm), with simulated cutting-zone temperatures overestimated by approximately 10 % in the absence of coolant/lubrication; this overestimation is fully compensated for under practical cooling conditions. These findings confirm that, for a given circumferential speed, feed velocity, and depth of cut, the proposed algorithm correctly predicts both thermal and vibrational behavior, allowing process engineers to maintain surface integrity and avoid thermal damage by reducing peripheral speed when necessary.

The analytical framework also incorporates surface-roughness and cutting-zone temperature checks: should calculated values exceed allowable limits (for instance, the allotropic transformation threshold of titanium, which begins as low as 450 °C and peaks around 880…900 °C), the peripheral speed is reduced to maintain surface integrity and prevent thermal damage. In practice, this means that for a given depth of cut (e.g., $t_c = 0.001$ cm) and cutting speed, one can determine in advance whether the resulting temperature (e.g., 970 °C in Ti-6Al-4V without coolant) or micro-roughness (e.g., 0.31 μm) will exceed acceptable thresholds; if so, process parameters are adjusted to ensure that processed surfaces remain burn-free and within tolerance. Experimental oscillograms of the cutting forces and vibration amplitudes demonstrated that, although the oscillation periods of the cutting forces and of the MFTW-system are close, resonance can be prevented by the strategies outlined, while the Saint-Venant–Boussinesq approximation ensures accurate amplitude predictions (deviations < 9%).

Finally, based on the combined theoretical and empirical findings, concrete recommendations have been formulated for selecting grinding regimes and design parameters (fixture compliance, tool geometry, workpiece mass) that ensure process stability and minimize the formation of thermal and vibration-induced defects. Specifically, practitioners are advised to:

– calculate the natural frequency ($\omega_0$) and damping coefficient ($\delta$) of the MFTW-system using workpiece geometry (length *L*, height *H*, width *B*), material density (ρ), and Young's modulus (*E*),

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

then determine resonant zones by matching $\omega_0$ to the dominant frequencies of the cutting-force oscillations;

– adjust cutting speeds ($v_c$) and depths of cut ($t_c$) such that the maximum cutting force ($F_{\max}$) does not coincide with the system's natural frequency; if resonance is detected, a 10…15 % phase shift or amplitude reduction suffices to suppress it;

– incorporate fixture-clamping compliance ($C_f$) and damping constant ($\kappa$) into the model to account for the real dynamic behavior of the machine–fixture interface; this addition reduces the theoretical $F_{max}$ by approximately 15…20 % to match experimental values;

– continuously monitor cutting-zone temperature ($T_{cz}$) and surface micro-roughness ($R_z$) through analytical formulas – e.g., for steel 102Cr6, $R_z \approx$ 0.0005…0.0015 µm at feed rates $\omega_f$ = 17.74…26.19 m/s – and compare to allowable limits (e.g., 700…770 °C for X12CrNiTi18-9). When $T_{cz}$ or $R_z$ approaches its limit, reduce vc by 5…10 % or apply enhanced cooling to maintain stability.

– select diamond grinding wheels with appropriate radius ($R$), number of active grains ($N_r$), and engagement length ($L_z$) to ensure that the total cutting force $F(\tau)$ remains within the elastic range of the workpiece, avoiding plastic deformation that would increase roughness and defect formation.

In summary, by integrating dynamic modeling of the MFTW-system with thermal and surface-integrity checks, the proposed approach furnishes a comprehensive strategy for optimizing finishing operations on materials susceptible to thermal defects. Its successful implementation in both steels (102Cr6, X12CrNiTi18-9) and titanium alloys confirms its wide applicability, offering a reliable toolset for practitioners seeking to balance material removal rates, surface quality, and equipment longevity in precision grinding processes.

**Література**

1. Assessment of surface structure of machined surfaces / Dobrocký D., Studený Z., Pokorný Z. et al. *Manufacturing Technology*. 2019. Vol. 19, Issue 4. P. 563–572. DOI:10.21062/UJEP/335.2019/A/1213-2489/MT/19/4/563.
2. Richert M., Dudek M., Sala D. Surface Quality as a Factor Affecting the Functionality of Products Manufactured with Metal and 3D Printing Technologies. *Materials*. 2024. Vol. 17, Issue 21. DOI:10.3390/MA17215371.
3. Aslan D., Budak E. Surface roughness and thermo-mechanical force modeling for grinding operations with regular and circumferentially grooved wheels. *Journal of Materials Processing Technology*. 2015. Vol. 223, 09.2015. P. 75–90. DOI:10.1016/J.JMATPROTEC.2015.03.023.
4. Analysis and prediction of surface integrity in machining: A review / Gao Y., Huang X., Lin M. J. et al. *Applied Mechanics and Materials*. 2014. Vol. 610, 2014. P. 1002–1020. DOI:10.4028/WWW.SCIENTIFIC.NET/AMM.610.1002.
5. Mitsyk A., Fedorovich V., Grabchenko A. Interaction of the abrasive medium with the treated surface and the process of metal removal during vibration treatment in the presence of a chemically active solution. *Cutting & Tools in Technological System.* 2021. Vol. 94. P. 42–48. DOI:10.20998/2078-7405.2021.94.05.
6. Toan N. K., Mezin N. Study of the Efficiency of Ultrasonic Turning of Heat-Resistant Alloys with Tools from Mineral Ceramics. *MATEC Web of Conferences*. 2019. Vol. 297, 2019. 01011. DOI:10.1051/MATECCONF/201929701011.
7. Clough R. W., Johnson C. P. A finite element approximation for the analysis of thin shells. International Journal of Solids and Structures. 1968. Vol. 4, Issue 1. P. 43–60. DOI:10.1016/0020-7683(68)90032-2.
8. Gurey V. I. Simulation of amplitude-frequency characteristics of friction strengthening process of flat machine parts' surfaces. *Bulletin of the National Technical University «KhPI» Series: Dynamics and Strength of Machines.* 2020. Vol. 2, P. 30–35. DOI:10.20998/2078-9130.2020.2.219636.
9. O. M. Serenko, I. V. Zakharova. Detection of residual stresses in coatings obtained by the method of arc metallization with a pulsating spraying flow. *Science and Transport Progress*. 2021. Vol. 5, Issue 89. P. 93–104. DOI:10.15802/stp2020/217611.
10. Strelchuk R. M., Trokhimchuk S. M. Mathematical modeling of the surface roughness of the grinding wheel during straightening. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu.* 2021. Vol. 1. P. 53–59. DOI:10.33271/NVNGU/2021-1/053.

11. Wear grinding wheels with precise working surface / Yakimov O., Bovnegra L., Uminsky S. et al. *Cutting & Tools in Technological System.* 2020. Vol. 92, 01.07.2020. P. 197–207. DOI:10.20998/2078-7405.2020.92.21.
12. Usov A., Kunitsyn M., Klymenko D., Davydiuk V. Modeling the effect of stochastic defects formed in products during machining on the loss of their functional dependencies. *Proceedings of Odessa Polytechnic University*. 2022. Vol. 1, Issue 65. P. 16–29. DOI:10.15276/opu.1.65.2022.02.
13. Possible ways to reduce the cost of grinding process of products with intermittent circles / Yakimov O., Zadorozhko G., Bovnegra L. et al. *Proceedings of Odessa Polytechnic University*. 2018. Vol. 1, Issue 54. P. 36–45. DOI:10.15276/opu.1.54.2018.05.
14. About effective use of wheels with intermittent work surface / Uminsky S., Yakimov O., Bovnegra L. et al. *Proceedings of Odessa Polytechnic University*. 2018. Vol. 2, Issue 55. P. 19–27. DOI:10.15276/opu.2.55.2018.02.
15. Usov A. V., Yakimov A. A. Parametric resonances arising from the piecewise constant excitation of elastic system of the grinding machine. *Proceedings of Odessa Polytechnic University*. 2014. Vol. 1, Issue 43. P. 60–68. DOI:10.15276/opu.1.43.2014.11.
16. Pyzsov I., Fedorovich V., Voloshkina I. The peculiarities of setting the grinding coefficient in the processing of polycrystalline superhard materials. *Cutting & Tools in Technological System*. 2020. Vol. 92. P. 170–178. DOI:10.20998/2078-7405.2020.92.18.
17. Rudyk A. V., Chupryna V. M., Pasov G. V., Venzhega V. I. Methods for determining the efficiency of the grinding process. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*. 2020. Vol. 5. P. 95–100. DOI:10.33271/NVNGU/2020-5/095.
18. Dyadya S., Kozlova O., Kushnir E., Karamushka, D. Research of oscillations during end-milling and their influence on the formation of the machined surface. *Innovative Materials and Technologies in Metallurgy and Mechanical Engineering*. 2023. Vol. 2, 09.01.2023. P. 71–77. DOI:10.15588/1607-6885-2022-2-12.
19. Oleksik M., Dobrotă D., Tomescu M., Petrescu, V. Improving the performance of steel machining processes through cutting by vibration control. *Materials.* 2021. Vol. 14, Issue 19. DOI:10.3390/MA14195712.
20. Zagórski K., Kudelski R., Cieślik J. Research on the possibilities of improving the surface quality by using vibrations of a turning tool. *AIP Conference Proceedings*. 2020. Vol. 2239, 05.2020. DOI:10.1063/5.0008410.
21. Boley B. A., Weiner J. H. Theory of thermal stresses. Courier Corporation, 2012. 610 p.
22. Rysiński J., Drobina R., Tomaszewski J. Probability of the critical length of a fatigue crack occurring at the tooth foot of cylindrical geared wheels of the drive system of a fiomax 2000 ring spinner. *Fibres and Textiles in Eastern Europe.* 2017. Vol. 25. P. 134–144. DOI:10.5604/12303666.1227895.
23. Mabrouki T., Hamdi H., Salvatore F. Analytical modelling of material removal process in the case of orthogonal cutting and worn tools. *International Journal of Machining and Machinability of Materials*. Vol. 19, Issue 6. P. 570. DOI:10.1504/IJMMM.2017.10009890.
24. Moreno E., Cobo A., Palomo G., González M. N. Mathematical models to predict the mechanical behavior of reinforcements depending on their degree of corrosion and the diameter of the rebars. *Construction and Building Materials.* 2014. Vol. 61, 06.2014. P. 156–163. DOI:10.1016/J.CONBUILDMAT.2014.03.003.
25. Le Dref J., Landon Y., Dessein G., Espinosa C. Modelling kinematics and cutting forces in vibration assisted drilling. *Mechanics and Industry*. 2016. Vol. 17, Issue 3. DOI:10.1051/MECA/2015078.
26. Oniţă G. V., Brîndaşu P. D. Experimental study of vibrations in face milling cutting. Applied Mechanics and Materials. 2014. Vol. 657. P. 68–72.
27. Zrazhevsky G., Zrazhevska V. Obtaining and investigation of the integral representation of solution and boundary integral equation for the non-stationary problem of thermal conductivity. EUREKA: Physics and Engineering. 2016. Vol. 6, 11. P. 53–58. DOI:10.21303/2461-4262.2016.00216.
28. Оборский Г. А., Дащенко А. Ф., Усов А. В., Дмитришин Д.В. Моделирование систем: монография. Одесса : Астропринт, 2013. 664 р.
29. Burek J., Flejszar R., Jamuła B. Numerical simulation of cutting layer in internal corners milling. *Mechanik.* 2019. Vol. 7. P. 412–414. DOI:10.17814/MECHANIK.2019.7.46.
30. Biró I., Czampa M., Szalay T. Experimental model for the main cutting force in face milling of a high strength structural steel. *Periodica Polytechnica Mechanical Engineering*. 2015. Vol. 59, Issue 1. P. 16–22. DOI:10.3311/PPME.7516.
31. Mileșan M., Gîrdu C. C., Cîrtînă L., Rădulescu, C. Mathematical modelling study of Hardox400 steel parts' roughness and hardness, cut with CO2 laser. *Strojniski Vestnik/Journal of Mechanical Engineering*. 2020. Vol. 66, Issue 2. P. 127–141. DOI:10.5545/SV-JME.2019.6320.
32. A Model and Application of Vibratory Surface Grinding / Bechcinski G., Ewad H., Tsiakoumis V. et al. *Journal of Manufacturing Science and Engineering*. 2018. Vol. 140, Issue 10. DOI:10.1115/1.4040725.

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

**References**


1. Dobrocký, D., Studený, Z., Pokorný, Z., Joska, Z., & Faltejsek, P. (2019). Assessment of surface structure of machined surfaces. *Manufacturing Technology*, 19(4), 563–572. DOI: https://doi.org/10.21062/UJEP/335.2019/A/1213-2489/MT/19/4/563.
2. Richert, M., Dudek, M., & Sala, D. (2024). Surface Quality as a Factor Affecting the Functionality of Products Manufactured with Metal and 3D Printing Technologies. *Materials*, 17(21). DOI: https://doi.org/10.3390/MA17215371.
3. Aslan, D., & Budak, E. (2015). Surface roughness and thermo-mechanical force modeling for grinding operations with regular and circumferentially grooved wheels. *Journal of Materials Processing Technology*, 223, 75–90. DOI: https://doi.org/10.1016/J.JMATPROTEC.2015.03.023.
4. Gao, Y., Huang, X., Lin, M. J., Wang, Z. G., & Sun, R. L. (2014). Analysis and prediction of surface integrity in machining: A review. *Applied Mechanics and Materials*, 610, 1002–1020. DOI: https://doi.org/10.4028/WWW.SCIENTIFIC.NET/AMM.610.1002.
5. Mitsyk, A., Fedorovich, V., & Grabchenko, A. (2021). Interaction of the abrasive medium with the treated surface and the process of metal removal during vibration treatment in the presence of a chemically active solution. *Cutting & Tools in Technological System*, 94, 42–48. DOI: https://doi.org/10.20998/2078-7405.2021.94.05.
6. Toan, N. K., & Mezin, N. (2019). Study of the Efficiency of Ultrasonic Turning of Heat-Resistant Alloys with Tools from Mineral Ceramics. *MATEC Web of Conferences*, 297, 01011. DOI: https://doi.org/10.1051/MATECCONF/201929701011.
7. Clough, R. W., & Johnson, C. P. (1968). A finite element approximation for the analysis of thin shells. *International Journal of Solids and Structures*, 4(1), 43–60. DOI: https://doi.org/10.1016/0020-7683(68)90032-2.
8. Gurey, V. I. (2020). Simulation of amplitude-frequency characteristics of friction strengthening process of flat machine parts' surfaces. *Bulletin of the National Technical University «KhPI» Series: Dynamics and Strength of Machines*, 2, 30–35. DOI: https://doi.org/10.20998/2078-9130.2020.2.219636.
9. Serenko, O. M. & Zakharova, I. V. (2021). Detection of residual stresses in coatings obtained by the method of arc metallization with a pulsating spraying flow. *Science and Transport Progress*, 5(89), 93–104. DOI: https://doi.org/10.15802/stp2020/217611.
10. Strelchuk, R. M., & Trokhimchuk, S. M. (2021). Mathematical modeling of the surface roughness of the grinding wheel during straightening. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 1, 53–59. DOI: https://doi.org/10.33271/NVNGU/2021-1/053.
11. Yakimov, O., Bovnegra, L., Uminsky, S., Tonkonogyi, V., & Shichireva, Y. (2020). Wear grinding wheels with precise working surface. *Cutting & Tools in Technological System*, 92, 197–207. DOI: https://doi.org/10.20998/2078-7405.2020.92.21.
12. Usov, A., Kunitsyn, M., Klymenko, D., & Davydiuk, V. (2022). Modeling the effect of stochastic defects formed in products during machining on the loss of their functional dependencies. *Proceedings of Odessa Polytechnic University*, 1(65), 16–29. DOI: https://doi.org/10.15276/opu.1.65.2022.02.
13. Yakimov, O., Zadorozhko, G., Bovnegra, L., Beznos, S., & Balan, V. (2018). Possible ways to reduce the cost of grinding process of products with intermittent circles. *Proceedings of Odessa Polytechnic University*, 1(54), 36–45. DOI: https://doi.org/10.15276/opu.1.54.2018.05.
14. Uminsky, S., Yakimov, O., Bovnegra, L., Klimenko, N., & Balan, V. (2018). About effective use of wheels with intermittent work surface. *Proceedings of Odessa Polytechnic University*, 2(55), 19–27. DOI: https://doi.org/10.15276/opu.2.55.2018.02.
15. Usov, A. V., & Yakimov, A. A. (2014). Parametric resonances arising from the piecewise constant excitation of elastic system of the grinding machine. *Proceedings of Odessa Polytechnic University*, 1(43), 60–68. DOI: https://doi.org/10.15276/opu.1.43.2014.11.
16. Pyzsov, I., Fedorovich, V., & Voloshkina, I. (2020). The peculiarities of setting the grinding coefficient in the processing of polycrystalline superhard materials. *Cutting & Tools in Technological System*, 92, 170–178. DOI: https://doi.org/10.20998/2078-7405.2020.92.18.
17. Rudyk, A. V., Chupryna, V. M., Pasov, G. V., & Venzhega, V. I. (2020). Methods for determining the efficiency of the grinding process. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, 5, 95–100. DOI: https://doi.org/10.33271/NVNGU/2020-5/095.
18. Dyadya, S., Kozlova, O., Kushnir, E., & Karamushka, D. (2023). Research of oscillations during end-milling and their influence on the formation of the machined surface. *Innovative Materials and Technologies in Metallurgy and Mechanical Engineering*, 2, 71–77. DOI: https://doi.org/10.15588/1607-6885-2022-2-12.
19. Oleksik, M., Dobrotă, D., Tomescu, M., & Petrescu, V. (2021). Improving the performance of steel machining processes through cutting by vibration control. *Materials*, 14(19). DOI: https://doi.org/10.3390/MA14195712.

20. Zagórski, K., Kudelski, R., & Cieślik, J. (2020). Research on the possibilities of improving the surface quality by using vibrations of a turning tool. *AIP Conference Proceedings*, 2239. DOI: https://doi.org/10.1063/5.0008410.
21. Boley, B. A., & Weiner, J. H. (2012). Theory of thermal stresses. Courier Corporation.
22. Rysiński, J., Drobina, R., & Tomaszewski, J. (2017). Probability of the critical length of a fatigue crack occurring at the tooth foot of cylindrical geared wheels of the drive system of a fiomax 2000 ring spinner. *Fibres and Textiles in Eastern Europe*, 25, 134–144. DOI: https://doi.org/10.5604/12303666.1227895.
23. Mabrouki, T., Hamdi, H., & Salvatore, F. (2017). Analytical modelling of material removal process in the case of orthogonal cutting and worn tools. International *Journal of Machining and Machinability of Materials*, 19(6), 570. DOI: https://doi.org/10.1504/IJMMM.2017.10009890.
24. Moreno, E., Cobo, A., Palomo, G., & González, M. N. (2014). Mathematical models to predict the mechanical behavior of reinforcements depending on their degree of corrosion and the diameter of the rebars. *Construction and Building Materials*, 61, 156–163. DOI: https://doi.org/10.1016/J.CONBUILDMAT.2014.03.003.
25. Le Dref, J., Landon, Y., Dessein, G., & Espinosa, C. (2016). Modelling kinematics and cutting forces in vibration assisted drilling. *Mechanics and Industry*, 17(3). DOI: https://doi.org/10.1051/MECA/2015078.
26. Oniţă, G. V., & Brîndaşu, P. D. (2014). Experimental study of vibrations in face milling cutting. *Applied Mechanics and Materials*, 657, 68–72. DOI: https://doi.org/10.4028/WWW.SCIENTIFIC.NET/AMM.657.68.
27. Zrazhevsky, G., & Zrazhevska, V. (2016). Obtaining and investigation of the integral representation of solution and boundary integral equation for the non-stationary problem of thermal conductivity. *EUREKA: Physics and Engineering*, 6, 53–58. DOI: https://doi.org/10.21303/2461-4262.2016.00216.
28. Oborsky, G. A., Dashchenko, A. F., Usov, A. V., & Dmitrishin, D. V. (2013). *Modeling of systems.* Odessa: Astroprint.
29. Burek, J., Flejszar, R., & Jamuła, B. (2019). Numerical simulation of cutting layer in internal corners milling. *Mechanik*, 7, 412–414. DOI: https://doi.org/10.17814/MECHANIK.2019.7.46.
30. Biró, I., Czampa, M., & Szalay, T. (2015). Experimental model for the main cutting force in face milling of a high strength structural steel. *Periodica Polytechnica Mechanical Engineering*, 59(1), 16–22. DOI: https://doi.org/10.3311/PPME.7516.
31. Mileșan, M., Gîrdu, C. C., Cîrtînă, L., & Rădulescu, C. (2020). Mathematical modelling study of Hardox400 steel parts' roughness and hardness, cut with CO2 laser. *Strojniski Vestnik/Journal of Mechanical Engineering*, 66(2), 127–141.
32. Bechcinski, G., Ewad, H., Tsiakoumis, V., Pawlowski, W., Kepczak, N., McMillan, A., & Batako, A. D. L. (2018). A Model and Application of Vibratory Surface Grinding. *Journal of Manufacturing Science and Engineering*, 140(10). DOI: https://doi.org/10.1115/1.4040725.

**Усов Анатолій Васильович**; Anatolii Usov, ORCID: https://orcid.org/0000-0002-3965-7611
**Куніцин Максим Володимирович**; Maksym Kunitsyn, ORCID: https://orcid.org/0000-0003-1764-8922

Received May 04, 2025

Accepted June 12, 2025