ISSN 2076-2429 (print)
ISSN 2223-3814 (online)
Proceedings of Odessa Polytechnic University, Issue 1(71), 2025

UDC 621.9.025.7

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CONTROL OF THE MILLING MODE OF A THIN-WALLED CONSOLE PLATE BY INDIRECT INDICATORS

А. Ткач, І. Сидоренко, І. Прокопович, В. Курган, А. Торопенко. **Управління режимом фрезерування тонкостінної консольної пластини за непрямими показниками.** У статті представлено теоретичне дослідження зміни прогину тонкої прямокутної пластини в залежності від положення точки застосування сили, що імітує навантаження від фрези. Моделювання проведено для двох характерних траєкторій обробки – вздовж довжини та вздовж ширини пластини. Розраховано прогини та відповідні їм значення жорсткості, отримано залежності та інтерпретовано їх особливості. Встановлено, що прийнятий алгоритм проведеного розрахунку аналогічний до алгоритмів розрахунку, які застосовуються в існуючих CAE системах. Виходячи з цього запропоновано використовувати дані, зокрема величину прогину пластини, як непрямий показник для управління режимом фрезерування. Прийняті як вихідні непрямі показники апроксимовані у вигляді статечного полінома з певною достовірністю апроксимації R2. Запропоновано використовувати отримані поліноми для створення відповідних зворотно пропорційних поліномів, що визначають один з основних показників режиму фрезерування – лінійну подачу шпинделя. Проведено програмну реалізацію запропонованого рішення з використанням розширеного G-коду, що передбачає застосування запропонованого рішення для широкої номенклатури фрезерних верстатів з ЧПУ. Проведено практичну апробацію отриманих програмних рішень, що підтвердили їхню повну працездатність. Результати досліджень дозволяють підвищити стійкості фрезерного процесу при обробці маложорстких, консольно закріплених деталей на верстатах без активної системи управління, що включає активний зворотний зв'язок «деталь – інструмент – верстат». Такий підхід дозволяє використовувати більш дешеві верстати отримуючи при цьому прийнятну якість оброблених поверхонь, що, у свою чергу, визначає зниження собівартості продукції, що випускається, і підвищує ефективність виробництва.
Ключові слова: фрезерування, жорсткість, прогин, тонка пластина, траєкторія обробки

A. Tkach, I. Sydorenko, I. Prokopovych, V. Kurhan, A. Toropenko. **Control of the milling mode of a thin-walled console plate by indirect indicators.** The article presents a theoretical study of the change in the deflection of a thin rectangular plate depending on the position of the point of application of force simulating the load from the cutter. The modeling was carried out for two typical processing trajectories – along the length and along the width of the plate. The deflections and the corresponding stiffness values are calculated, the dependencies are obtained and their features are interpreted. It is found that the adopted algorithm of the calculation is similar to the calculation algorithms used in existing CAE systems. Based on this, it is proposed to use the data, in particular the plate deflection value, as an indirect indicator for controlling the milling mode. The indirect indicators adopted as the initial ones are approximated as a power polynomial with a certain approximation reliability R2. It is proposed to use the obtained polynomials to create the corresponding inversely proportional polynomials that determine one of the main indicators of the milling mode – the linear feed of the spindle. The software implementation of the proposed solution was carried out using an extended G-code, which suggests the use of the proposed solution for a wide range of CNC milling machines. Practical testing of the obtained software solutions was carried out, confirming their full operability. The research results allow increasing the stability of the milling process when processing low-rigid cantilever-fixed parts on machines without an active control system, including active feedback "part – tool – machine". This approach allows using cheaper machines while obtaining acceptable quality of machined surfaces, which, in turn, determines a decrease in the cost of manufactured products and increases production efficiency.
Keywords: milling, rigidity, deflection, thin plate, processing trajectory

1. Introduction

The processing of thin-walled parts by milling is often accompanied by vibrations and loss of stability, which negatively affects the surface quality and the accuracy of the geometry of the workpiece. One of the reasons for the occurrence of these vibrations and loss of stability are self-oscillations, namely undamped oscillations in the "machine – tool – workpiece" system, which are maintained by the machine's energy source, and their amplitude and frequency are determined by the properties, usually the rigidity, of the system itself. Reducing the negative manifestation of this type of oscillation is especially important when processing the ends of thin plates, in the case when the cutter moves along the free edge, since this phenomenon leads not only to deterioration in the quality of the surface being processed, but also to damage to the tool or workpiece.

Taking into account modern trends of expanding the use of CNC milling machines for performing such technological operations, the analysis of changes in rigidity along the processing path is important for assessing the conditions of stable processing and determines methods for suppressing self-

DOI: 10.15276/opu.1.71.2025.04

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

oscillations in the form of: optimizing cutting modes; increasing the rigidity of the system; using active and passive methods of vibration damping; using a special tool.

2. Literature analysis

Research devoted to milling thin-walled parts and their thin-walled elements primarily focuses on problems associated with the occurrence of vibrations due to changes in rigidity during processing. This is due to both the geometric features of the parts and the conditions of fastening and cutting modes [1, 2].

One of the common ways to improve machining accuracy is to modify CNC control algorithms, which allows taking into account the behavior of the tool and the workpiece in real time. Such approaches allow compensating for interpolation errors and increasing the stability of the process [1, 3]. Some studies demonstrate that vibration activity can be significantly reduced by selecting rational feed parameters and cutting depth, especially during contour machining of thin walls [3, 4].

In this regard, mathematical modeling of the milling process taking into account changes in rigidity that determine vibration plays a key role in predicting shape deviations [3]. At the same time, it is necessary to take into account not only the cutting forces, but also the conditions for fixing the part, especially with asymmetric or cantilever fastening [3, 5]. Existing methods for analyzing self-oscillations during technological processing in the form of milling, based on the time-frequency characteristics of signals, in the case of using active analysis and control systems, make it possible to identify zones of unstable operation and correct these technological modes [4, 6]. A promising direction for correcting technological modes is the use of two-dimensional models of plate bending, which make it possible to more accurately take into account the distribution of stresses and strains over the entire surface of the part [7]. In this case, special attention is paid to the boundary conditions and geometry of the processed zone, which is critical for precision processing.

Practical examples demonstrate that the rigidity of a structure can be significantly increased through rational spatial configuration or additional technological techniques, such as temporary double-sided bending during processing [8, 9].

Topological optimization of variable-thickness plates is also an effective method for reducing vibrations. It ensures the redistribution of material in such a way as to minimize the compliance of the structure in the zones of maximum loads [10]. A number of studies are aimed at developing models that allow for large deflections and nonlinear behavior of thin plates. Such models are especially relevant for high-precision processing or work with composite materials, where linear approaches give significant errors [11, 12].

An important role is played by the analysis of vibration signals during milling, allowing for the rapid detection of unstable modes and correction of the tool trajectory. Such methods are especially useful when milling parts with variable thickness or variable rigidity conditions [13].

Special attention is paid to the frequency analysis of the occurrence of self-oscillations, especially during the transition to modes with bifurcation and loss of stability. This allows us to identify potentially dangerous zones in the frequency range at the modeling stage and exclude them from the process route [10].

However, the indicated approaches to reducing the negative manifestation of vibrations during milling can be used only in the case of using CNC machines equipped with rather expensive and complex in manufacturing and operation active control systems with sensors and feedback, defining the active control system “part – tool – machine”.

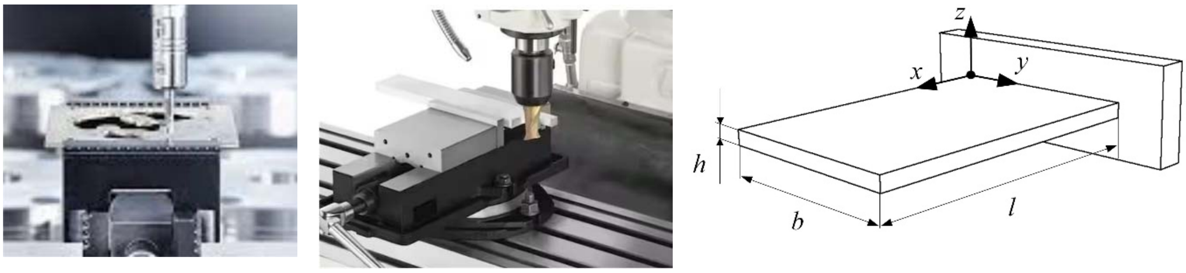
Purpose of research

Considering that the number of CNC milling machines is constantly expanding, and the vast majority of them do not have active analysis and control systems due to the requirements for reducing their cost, it is of interest to solve the problems of cutting mode control without using the feedback “part – tool – machine”. It is proposed to solve this problem by controlling the cutting modes of the CNC machine using indirect indicators in the form of elastic deformation (deflection) of the workpiece, which can be obtained on the basis of computer modeling of this part using CAM and CAE tools.

The practical significance of this work lies in the possibility of adapting the research results to configure a wide range of “inexpensive” CNC machines and designing technological processes for processing low-rigidity parts that determine acceptable quality.

3. Research method

Let us consider possible elastic deformations and the method of their determination as applied to the end milling operation of a part, considering it as a thin-walled element. With a certain configuration of a thin-walled part or the method of its fastening, some surfaces processed on it represent either an internal cantilever surface (Fig.1, *a*) or an external cantilever surface (Fig. 1, *b*).



a *b* *c*

Fig. 1. Operation of milling the end face of a thin-walled element: internal cantilever surface (*a*); external cantilever surface (*b*); model of the cantilever surface (*c*)

When modeling, such a surface, with certain simplifications, can be considered as a cantilever beam of rectangular cross-section (Fig. 1, *c*).

The moment of inertia of the cross-section of the beam under consideration is calculated according to the expression:

$$I = \frac{(b \cdot h^3)}{12}, \tag{1}$$

where: I – moment of inertia about the neutral axis; b – width of the cross-section (horizontal dimension); h – height (vertical dimension, perpendicular to the direction of bending); 12 – result of integration y^2dA over the area of the rectangle to take into account the distribution of mass (or area) about the axis.

Accordingly, the deflection of a cantilever beam with a concentrated load at its free end is calculated as:

$$w = \frac{(F \cdot l^3)}{(3 \cdot E \cdot I)}, \tag{2}$$

where: F – force applied to the free end of the beam; l – length of the beam from the point of attachment to the point of application of the force; E – modulus of elasticity of the material (young's modulus); I – moment of inertia of the beam section relative to the neutral axis.

In the case of load displacement along the length (along the beam x-axis, according to the adopted coordinate system (Fig. 1, *c*), the deflection at an arbitrary point can be calculated as:

$$w(x) = \frac{F \cdot x^2}{6 \cdot E \cdot I}(3 \cdot l - x), \tag{3}$$

where: F – concentrated force applied to the plate; x – distance from the fixed end of the beam to the point at which the deflection is calculated; F – force applied to the free end of the beam; E – modulus of elasticity of the material (young's modulus); I – moment of inertia of the beam cross-section; l – length of the cantilever beam (the distance from the fixation to its free end).

In the case of load displacement across the width of the plate (along the beam y-axis, according to the adopted coordinate system (Fig. 1, *c*), the deflection at an arbitrary point can be calculated as:

$$w(y) = \frac{F \cdot y^2}{(6 \cdot E \cdot I)}(3 \cdot b - y), \tag{4}$$

where: F – concentrated force applied to the plate; E – elastic modulus of the material; I – moment of inertia of the cross section; b – length of the side under consideration (along the y axis); here is the width of the plate; y – distance from the fixed edge along the width; 6 is the normalizing factor from the solution of the bending problem; 3 – factor associated with the change in curvature along the length of the beam.

In order to obtain the relationship between deflection and rigidity, one can use the expression in which rigidity is defined as the ratio of the force F to the corresponding deflection w:

Праці Одеського політехнічного університету, 2025. Вип. 1(71)
ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

$$k = \frac{F}{w}. \quad (5)$$

Thus, the behavior of the function $k(x)$ and $k(y)$ along the trajectory of the cutter is known. It is this algorithm for calculating the deflection that is implemented in the most common CAE systems.

4. Results

Based on the presented method of determining the deflection, the proposed approach, determining the control of the milling modes of a thin-walled flat rectangular plate on a CNC machine without active feedback, is based on the following. In the vast majority of cases, the control systems used in these machines allow changing only two interrelated parameters during operation, namely the feed F (mm/min) and the spindle speed S (1/min).

As an example, the system of part *1*, tool *2* and machine *3* is considered, describing the process of milling with controlled feed F of a thin rectangular plate made of Steel 45 with dimensions of 60 mm (length) × 40 mm (width) × 3 mm (thickness), rigidly fixed along one short edge (Fig. 2, *a*).

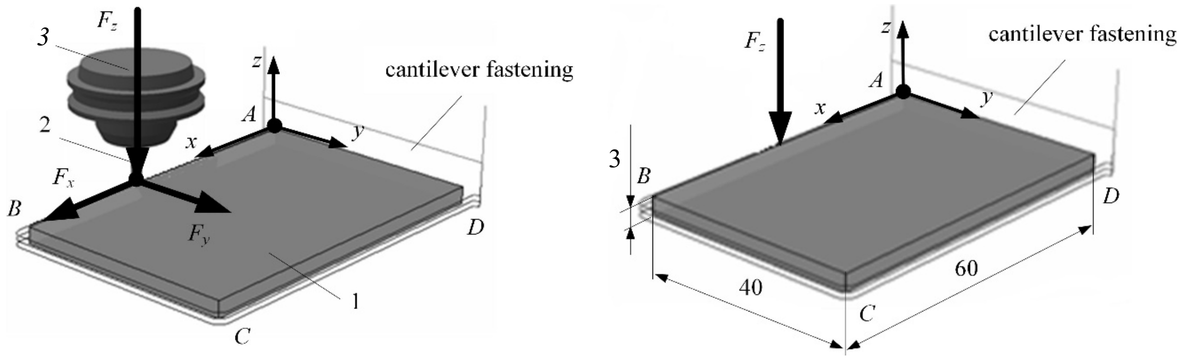


Fig. 2. Simulation of the milling process of a cantilever thin-walled plate: the "part – tool – machine" system (*a*); calculation scheme (*b*)

The main load determining the deflection is the concentrated force in the form of the axial cutting force F_z, at a nominal spindle speed of $S = 3600$ 1/min (Fig. 2, *b*). The calculation of this force is made according to the expression:

$$F_z = C_F \cdot t \cdot S_z^{yF} \cdot D^{uF} \cdot Z \cdot K_F, \quad (6)$$

where: C_F – coefficient depending on the workpiece material and cutting conditions (for Steel 45 when processed with a carbide cutter $C_F = 300$ (N/mm²) – specific cutting force); t – cutting depth (for the case under consideration $t = 3$ mm); S_z – feed per cutter tooth (for the case under consideration $S_z = 0.03$ mm); D – cutter diameter (for the case under consideration $D = 16$ mm); Z – number of cutter teeth (for the case under consideration $Z = 2$); K_F – correction coefficient ($K_F \approx 1$ is adopted).

Graphical interpretation of the calculation results using expressions (3) and (4) shows that when the cutter moves from point A to point B (along the x coordinate), the deflection smoothly increases and reaches a maximum at point B (Fig. 3, *a*).

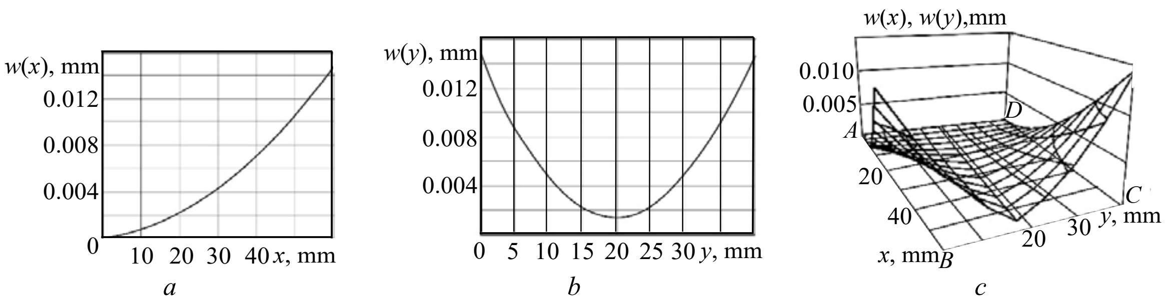


Fig. 3. Calculated value of deflection: by the adopted x coordinate (*a*); by the adopted coordinate y (*b*); three-dimensional interpretation (*c*)

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)
Proceedings of Odessa Polytechnic University, Issue 1(71), 2025

Similarly, when moving from point *B* to point *C* (along the *y* coordinate), the deflection is also maximum at the extreme points *B* and *C*, but has a certain minimum in the center of the section (Fig. 3, *b*). Thus, the deflection graph has the shape of a parabola with branches directed upward. Three-dimensional graphical interpretation of the calculation results indicates a complex stress-strain state of the plate during its milling with a constant feed (Fig. 3, *c*).As indicated above, the milling process is most unstable in areas with maximum deflection. It is at points *B* and *C* that self-oscillations may occur. The calculated decrease in deflection between these points indicates better stability in the central part of the movement along the *y* coordinate.

Analysis of the obtained results allows us to propose the following approach to feed correction. The change in deflection both along the $x - w(x)$ coordinate and along the $y - w(y)$ coordinate can be represented in the form of corresponding polynomial expressions with a certain approximation reliability R^2. For the case under consideration, the change in deflection along the *x* and *y* coordinates is represented by the expressions:

$$w(x) = 4 \cdot 10^{-6} x^2 + 2 \cdot 10^{-5} x - 6 \cdot 10^{-5}, \; R^2 = 0.9999;$$
$$w(y) = 3 \cdot 10^{-5} y^2 + 0.013y - 0.015, \; R^2 = 0.9881. \quad (6)$$

It is proposed to link the calculated deflections (6) with the cutting mode, thereby determining at the initial stage of the study an inversely proportional relationship between the feed in the $x-y$ directions and the corresponding deflections $w(x)$, $w(y)$ (Fig. 4).

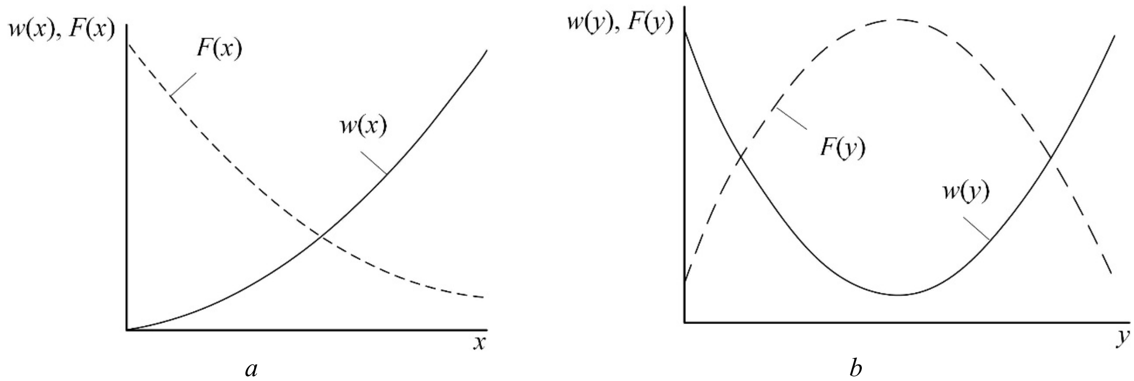


Fig. 4. Graphical interpretation of the proposed relationship between feed in the $x-y$ directions and the corresponding deviations $w(x) - w(y)$: along the *x* coordinate (*a*); along the y coordinate (*b*)

Based on the existing recommendations, it is accepted that the feed at the points of the processing path with minimum deflection (corresponds to coordinates ($x = 0$; $y = 0$) and ($x = 0$; $y = 40$) is maximum and equal to $F_{\max} = 500$ mm/min, and at the points with maximum deflection (corresponds to coordinates ($x = 60$; $y = 0$) and ($x = 60$; $y = 40$) it is minimum and equal to $F_{\min} = 50$ mm/min. At the same time, taking into account the proportionality adopted in the proposal, the corresponding polynomial expressions determining the corresponding feeds are obtained:

$$F(x) = [0.01464 + (3.95 \cdot 10^{-6} x^2 + 7 \cdot 10^{-6} x)] \cdot -1, \quad R^2 = 0.9789;$$
$$F(y) = [0.01464 + (3.5 \cdot 10^{-5} (x-20)^2 + 6.4 \cdot 10^{-4})] \cdot -1, \quad R^2 = 0.9881. \quad (7)$$

To check the feasibility of implementing the presented proposal, a practical software implementation of expressions (7) was carried out, oriented towards existing control systems of CNC machines. As an illustration, the corresponding control program is given, which was developed using an extended G-code with Siemens proprietary directives for the Sinumerik 840D sl milling machine (Fig. 5).

The conducted full-scale tests using the presented program, in which the trajectory of the machine spindle and the current feed rates (according to the machine sensors) were controlled, confirm its full operability. This allows us to proceed to the real milling process according to this program for subsequent determination of the practical purity of the end face processing of the part in question at any point of its processing trajectory.

Considering the base of the presented program in the form of an extended G-code, its analogue can also be implemented for Fanuk CNC machines.

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

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gcode
; 1. SYSTEM INITIALIZATION
    G90 G94 G54 G40 G17 G64                           ; G64 Smooth transitions
; 2. DETERMINING PARAMETERS
    PARAB_A = 0.000035                                ; For (POS+20)^2
    PARAB_B = 0.00064                                 ; Offset
    POLN_MAX = 0.01464                                ; Max. polynomial value
    F_MIN = 50  F_MAX = 600  F_RANGE = 550            ; Min. feed   Max. feed at X = 0   Feed range
    Y_START = 0  Y_END = 60                           ; Beginning and end of vertical section
    X_START = 0  X_END = -40                          ; Beginning and end of horizontal section
    VERTEX_OFFSET = 20  STEP = 0.5                    ; Parabola Vertex Shift   Step Shift (mm)
    Z_SAFE = 5  Z_CUT = -2                            ; Rapid feed height   Cutting depth
; 3. SAFE TOOL SUPPLY
    G0 X0 Y[Y_START] Z[Z_SAFE] G1 Z[Z_CUT] F300       ; Approaching the beginning of the section    Lowering
                                                        the tool
; 4. VERTICAL PROCESSING (Y0→Y60)
    Y_POS = Y_START
    WHILE Y_POS <= Y_END;                             ; Calculating the value of the polynomial (adapted for
                                                        Y)
    Y_SHIFTED = Y_POS - VERTEX_OFFSET
    POLN_VAL = PARAB_A * Y_SHIFTED * Y_SHIFTED +
    PARAB_B
    CURRENT_F = F_MIN + F_RANGE * (1 -                ; Scaling the feed
    POLN_VAL/POLN_MAX)
    CURRENT_F = MAX(F_MIN, MIN(F_MAX, CURRENT_F))
    G1 X0 Y[Y_POS] F[ROUND(CURRENT_F, 1)]             ; Moving
    Y_POS = Y_POS + STEP                              ; Processing step
    ENDWHILE
; 5. HORIZONTAL PROCESSING (X0→X-40)
    X_POS = X_START
    WHILE X_POS >= X_END
    X_SHIFTED = X_POS + VERTEX_OFFSET  ; X - (-20) = X +   ; Calculating the value of the polynomial (adapted for
    20                                                  X)
    POLN_VAL = PARAB_A * Y_SHIFTED * Y_SHIFTED +
    PARAB_B
    CURRENT_F = F_MIN + F_RANGE * (1 -                ; Scaling the feed
    POLN_VAL/POLN_MAX)
    CURRENT_F = MAX(F_MIN, MIN(F_MAX, CURRENT_F))
    G1 X[X_POS] Y[Y_END] F[ROUND(CURRENT_F, 1)]       ; Moving
    X_POS = X_POS - STEP                              ; Processing step
    ENDWHILE
; 6. FINAL OPERATIONS
    G0 Z[Z_SAFE]                                      ; Lifting the tool
    M30                                               ; End of program
```

Fig. 5. Developed and tested program for controlling the “Sinumerik 840D sl” milling machine when solving a research problem

It should be noted that the standard G-code (ISO 6983) does not implement special commands for raising to a power. In such cases, raising to an integer power can be implemented as a sequential multiplication. If the controller used allows it, then logarithms and exponents can be used for raising to a power. For the fairly popular CNC machine control software Mach3, such tasks are more rationally solved using macros/scripts (for example, in Mach3 via VBScript).

Conclusions

According to the results of the research, the following was established:

1. The development of CAD CAM CAE systems implies their closer interaction. The increased capabilities of CAE systems for analyzing complex-deformed state of objects when modeling various types of loading, including from a cutting tool, allows obtaining the results of corresponding calculations with their subsequent use as indirect indicators.

2. Indirect indicators obtained in CAE systems can be used as input parameters for CAM systems, determining the possibility of optimal processing of a part at the design stage.

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)
Proceedings of Odessa Polytechnic University, Issue 1(71), 2025

3. The proposed approach of using pre-determined indirect indicators of the stress-strain state in CAM systems allows in some cases to abandon the use of expensive active systems "part – tool – machine" on CNC machines and use cheaper machines while obtaining acceptable quality of processed surfaces, which determines a reduction in the cost of manufactured products and increases production efficiency.

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Received March 23, 2025

Accepted May 01, 2025