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MATHEMATICAL MODEL OF HEAT TRANSFER IN THE "FROZEN CASTING MOLD – ANTI-ADHESIVE COATING – METAL" SYSTEM BASED ON CELLULAR AUTOMATA

М. Замятін, Т. Лисенко, Ю. Морозов, В. Замятін, К. Крейцер. **Математична модель теплопереносу в системі «заморожена ливарна форма – протипригарне покриття – метал» на основі клітинних автоматів.** Розроблено математичну модель теплопереносу в системі «заморожена ливарна форма – протипригарне покриття – метал» на основі клітинних автоматів. Моделювання переносу теплової енергії здійснюється на базі спеціальної модифікації методу стохастичних збудливих клітинних автоматів (Stochastic Excitable Cellular Automata – SECA). На підставі математичної моделі проведено розрахунки теплопереносу в системі «заморожена ливарна форма – протипригарне покриття – метал». Для визначення температур клітинних автоматів на нульовому кроці чисельного експерименту для кожного з них задаються початкові значення температури, величини теплопровідності, теплоємності. Потім на кожному n-му часовому кроці обчислюється нове значення температури автомата з урахуванням теплових потоків з боку кожного сусіднього автомата.При аналізі тепломасопереносу необхідно враховувати витрати тепла на плавлення та випаровування води, а також виділення тепла при конденсації вологи. Теплоту плавлення льоду та випаровування води враховували шляхом збільшення питомої теплоємності форми в інтервалі температур плавлення.Проведені дослідження дозволили експериментально підтвердити адекватність математичної моделі теплопереносу в системі «заморожена ливарна форма – протипригарне покриття – метал» на основі клітинних автоматів. Дослідження проводилися за допомогою сплаву АК5М2 при литті в заморожени ливарні форми з протипригарним покриттям.Представлена номограма для визначення мінімальної товщини замороженого стрижня залежно від товщини стінки виливки та температури охолодження форми. Описано метод вибору параметрів за номограмою. Отримана математична залежність глибини прогріву форми (стрижня) до -5,0 °С від товщини стінки виливка. При цій температурі міцність замороженої форми (стрижня) порівнюється з міцністю піщано-глинистої форми.
Ключові слова: заморожена ливарна форма, клітинні автомати, теплообмінні процеси, протипригарне покриття

M. Zamiatin, T. Lysenko, Yu. Morozov, V. Zamiatin, K. Kreitser. **Mathematical model of heat transfer in the "frozen casting mold – anti-adhesive coating – metal" system based on cellular automata**. A mathematical model for heat transfer in a "frozen casting mold – anti-adhesive coating – metal" system has been developed based on cellular automata. The modeling of thermal energy transfer is carried out using a special modification of the Stochastic Excitable Cellular Automata (SECA) method. Based on this mathematical model, heat transfer calculations were performed for the "frozen casting mold – anti-adhesive coating – metal" system. To determine the temperatures of the cellular automata at the initial step of the numerical experiment, each automaton is assigned initial values for temperature, thermal conductivity, and heat capacity. Then, at each subsequent *n*-th time step, a new automaton temperature value is calculated, considering thermal fluxes from each neighboring automaton. When analyzing heat and mass transfer, it is crucial to account for heat consumption during the melting and evaporation of water, as well as heat release during moisture condensation. The heat of ice melting and water evaporation was considered by increasing the specific heat capacity of the mold within the melting temperature range. The conducted research experimentally confirmed the adequacy of the mathematical model for heat transfer in the "frozen casting mold – anti-adhesive coating – metal" system based on cellular automata. The investigations were performed using an AK5M2 alloy cast into frozen molds with an anti-adhesive coating. A nomogram is presented for determining the minimum thickness of the frozen core depending on the casting wall thickness and the mold cooling temperature. A method for selecting parameters using this nomogram is described. A mathematical dependency of the mold (core) heating depth to –5.0 °C on the casting wall thickness was derived. At this temperature, the strength of the frozen mold (core) is comparable to that of a sand-clay mold.
Keywords: frozen casting mold, cellular automata, heat exchange processes, anti-adhesive coating

1. Introduction

Casting into frozen casting molds (FCM) represents one of the most promising directions in modern foundry production. This method opens up new possibilities for obtaining castings with improved characteristics and enhanced production efficiency. A key feature of casting into FCMs is a significant increase in mold strength compared to traditional sand-clay molds. This is achieved by us-

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ing water as the primary binder, which forms a robust framework upon freezing.

Beyond enhanced strength, the use of FCMs offers several other crucial advantages. These include an improved structure, increased precision, and better geometric parameters of castings, as well as savings in molding materials. Importantly, it leads to a better environmental situation in foundry shops due to the elimination of harmful emissions into the atmosphere. Traditional casting methods often involve the release of toxic substances, negatively affecting worker health and the environment. In contrast, the FCM casting process is more environmentally friendly and safer.

However, despite all its advantages, the FCM casting process is quite complex and demands meticulous control of technological parameters. The thermal processes occurring within the "frozen casting mold – anti-adhesive coating – metal" system play a pivotal role in shaping casting quality. Improper management of these processes can lead to defects, reduced mechanical properties, and a deterioration in the appearance of the products.

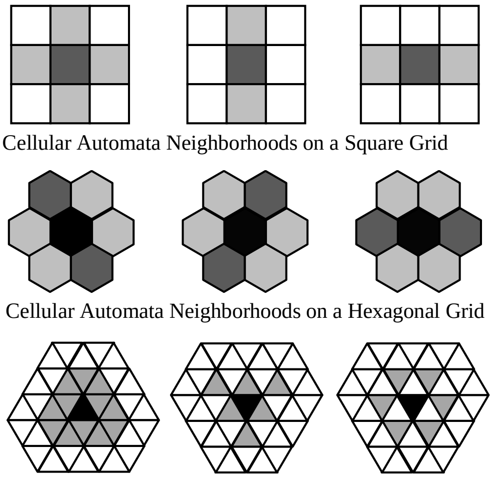

Fig. 1. Cellular Automata Neighborhoods

Consequently, the development of mathematical models that can accurately describe and predict heat transfer within the "frozen casting mold – anti-adhesive coating – metal" system is a relevant scientific and practical challenge. Such models can be utilized for optimizing technological processes, selecting optimal materials and casting regimes, and preventing defects in castings [1, 2].

2. Literature Review and Problem Statement

In this work, the modeling of thermal energy transfer is performed based on a special modification of the Stochastic Excitable Cellular Automata (SECA) method [3]. An excitable automaton is capable of performing a sequential chain of state switches under external influence. Each such automaton is characterized by a specific set of neighbors within the first coordination sphere (Fig. 1). It also possesses numerical parameters corresponding to the material contained within the modeled volume of space, such as dislocation density, specific thermal conductivity, specific heat capacity, coefficient of thermal expansion, etc [4, 5, 6].

During interaction with neighboring automata, the thermal and mechanical components of energy can change, and consequently, their associated physical parameters (temperature, entropy, stress, strain, density, etc.) (Fig. 2).

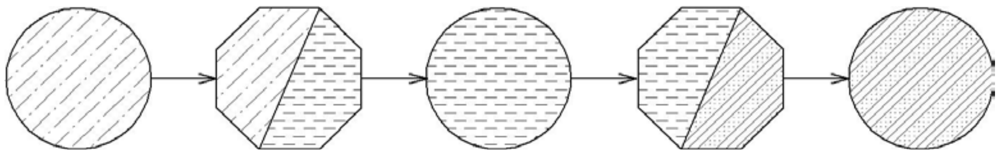

Fig. 2. Transition of a Cellular Automaton from One State to Another

Heat transfer is modeled according to Fourier's hypothesis, where the amount of heat dQ passing through an element of an isothermal surface dF during a time interval dx is directly proportional to the temperature gradient, i.e.:

$$\frac{dQ}{dFd\tau} = J_T = \lambda \mathrm{grad}(t), \tag{1}$$

where: J_T – is the heat flux density, defined as the amount of heat passing per unit time through a unit area of an isothermal surface; λ – is the thermal conductivity coefficient.

The causes that induce the flux J_i, i.e., temperature gradients, concentration gradients, external forces, etc., are commonly referred to as generalized forces and denoted as X_i.

To determine the temperatures of the cellular automata at the zero step of the numerical experiment, each of them is assigned initial values for temperature T_0, thermal conductivity λ, and heat ca-

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pacity *c*. Then, at each *n*-th time step, a new automaton temperature value is calculated, taking into account heat fluxes from each neighboring automaton:

$$T_i^n = T_i^{n-1} + \frac{1}{c_i} \cdot \sum_{k=1}^{n} Q_{ik}^n , \tag{2}$$

where: T_i^n, T_i^{n-1} – are the temperatures of the *i*-th cellular automaton at the *n*-th and (*n*−1)-th time steps, respectively; c_i– is the heat capacity of the *i*-th automaton; Q_{ik}^n – is the heat energy flux from the neighboring cellular automaton with index *k* into the considered automaton with index *i* at the *n*-th time step; *N* – is the number of neighbors.

The change in thermal energy Q_{ik}^n is calculated based on Fourier's law:

$$Q_{ik}^n = \frac{\lambda_{ik} \cdot \Omega}{l} \cdot (T_k^{n-1} - T_i^{n-1}) \cdot \Delta\tau , \tag{3}$$

where: λ_{ik} – is the mutual thermal conductivity coefficient; *l* – is the distance between the centers of the considered automata; Ω – is the area of the adjacent face; Δτ – is the time step value.

A cellular automaton can exist in three states: frozen mold, wet mold, and dry mold. Each of these states is characterized by different thermophysical properties (Table 1).

Table 1

Thermophysical Characteristics of the Molding Mixture [5]

Composition	ρ, kg/cm³	λ, W/(m·°C)	C, kJ/(kg·°C)
Sand+Ice	1614	1.16	2.1
Sand+Water	1619	1.08	2.19
Sand+Steam	1572	1.04	2.09

Also, when analyzing heat and mass transfer, it is necessary to consider the heat consumption for melting and evaporation of water, as well as the heat release during moisture condensation.

The heat of ice melting and water evaporation was accounted for by increasing the specific heat capacity of the mold c_2 within the melting temperature range:

$$c_2' c_2 + \frac{r_1 \cdot W \cdot T_i^n}{t_f^2} , \tag{4}$$

where: r_l – heat of fusion of ice, $r_l = 334000$ J/kg; *W* – humidity (or moisture content); *t* – temperature interval for melting (evaporation), °C:

$$c_2' = c_2 + \frac{r_p \cdot W \cdot T_i^n}{t_f^2} , \tag{5}$$

$r_p = 2257010$ J/kg – heat of water evaporation.

As steam penetrates deeper into the unheated mold, steam condensation occurs on colder sand grain surfaces, releasing heat of condensation and increasing the cellular automaton's temperature by ΔT_k:

$$\Delta T_k = \frac{r_p \cdot W}{c_2} , \text{°C}. \tag{6}$$

For a frozen mold, the cellular automaton's temperature equation is:

$$T_i^n = T_i^{n-1} + \frac{1}{c_i} \cdot \sum_{k=1}^{N} Q_{ik}^n . \tag{7}$$

For frozen and wet molds in the moisture condensation zone:

$$T_i^n = T_i^{n-1} + \frac{1}{c_i} \cdot \sum_{k=1}^{N} Q_{ik}^n + \Delta T_k = T_i^{n-1} + \frac{1}{c_i} \cdot \sum_{k=1}^{N} Q_{ik}^n + \frac{r_p W}{c_2} . \tag{8}$$

For a frozen mold during the melting period:

$$T_i^n = T_i^{n-1} + \frac{1}{c_i + \frac{r_l \cdot W \cdot T_i^n}{t_f^2}} \cdot \sum_{k=1}^{N} Q_{ik}^n . \tag{9}$$

For a wet mold during the moisture evaporation period:

$$T_i^n = T_i^{n-1} + \frac{1}{c_i + \frac{r_p \cdot W \cdot T_i^n}{t_f^2}} \cdot \sum_{k=1}^{N} Q_{ik}^n . \quad (10)$$

Switching between cellular automaton states occurs when 3 neighboring cellular automata adopt a higher state.

To determine the temperature at the “casting-mold” boundary, the following formula can be applied:

$$T_k = T_0 + (T_{pou} - T_0) \cdot \frac{b_c}{b_c + b_f}, \quad (11)$$

where: T_0 – initial mold temperature; T_{pou} – alloy temperature in the mold after pouring; b_c – heat accumulation coefficient of the alloy; b_f – heat accumulation coefficient of the mold.

The specific heat capacity of the mold c_f will be determined as an effective value, meaning it accounts for the heats of phase transformations of water during melting and evaporation:

$$c_f = \frac{\left(-c_c \cdot T_0 - c_1 \cdot T_0 + c_2 \cdot 100 + c_3 \cdot (T_k - 100) + \frac{W_0 \cdot (r_{ev} + r_{mel})}{(T_k - T_0)} \right)}{(T_k - T_0)}, \quad (12)$$

where: c_c, c_1, c_2, c_3 – respectively, the specific heat capacities of the anti-adhesive coating, frozen, wet, and dry molds; r_{ev}, r_{mel} – specific heat of evaporation and melting of water; W_0 – initial moisture content of the mixture, %.

When calculating c_f, we can assume:

$$T_k = T_{kr} = \frac{T_{liq} + T_{sol}}{2}, \quad (13)$$

where: T_{liq}, T_{sol} – liquidus and solidus temperatures of the alloy.

Similarly, to calculate the specific heat capacity of the alloy, the following formula can be applied:

$$c_c = c_{c1} + \frac{Q_{kr}}{T_{pou} - T_k}, \quad (14)$$

where Q_{kr} – specific heat of crystallization of the alloy; c_{c1} – specific heat capacity of the solid alloy.

Since in the initial moments after pouring, the melt directly contacts the frozen mold, and the dry layer thickness is infinitesimally small, as a first approximation, the following expression can be used to estimate the heat flux into the mold at the beginning of solidification:

$$dQ = \frac{b_f}{\sqrt{\pi\tau}} (T_k - T_0) F d\tau , \quad (15)$$

which is valid for a semi-infinite body under boundary conditions of the first kind.

Considering the above, we write the heat balance equation:

$$\frac{b_f}{\sqrt{\pi\tau}} (T_k - T_0) F d\tau = Q_{kr} \rho_c F d\varepsilon . \quad (16)$$

Integrating this equation, we obtain the formula for the kinetics of casting solidification in the initial moments of the process:

$$\varepsilon = \frac{2 b_f (T_k - T_0)}{\sqrt{\pi Q_{kr} \rho_c}} \sqrt{\tau} . \quad (17)$$

The value $m_1 = \frac{2 b_f (T_k - T_0)}{\sqrt{\pi Q_{kr} \rho_c}}$ is the solidification constant.

As seen from expression 17, an increase in b_f, which in turn grows with increased moisture content and decreased T_0, leads to accelerated solidification.

However, during the mold's heating process, the heat exchange conditions change significantly. An relatively low-thermal-conductive dry layer forms on its inner surface, and its thickness increases over time. The kinetics of casting solidification are then determined by the thermal resistance of this dry layer. Evaporated products released at the outer boundary of this layer move deeper into the mold, where they condense. To calculate the kinetics of the dry layer's growth under these conditions, the following formula can be applied:

$$\varepsilon = \sqrt{\frac{2n_2(n_2+1)a_3\tau}{1+\dfrac{(n_2+1)W_1 r_{ev}}{c_3(100-W_1)(T_{kr}-T_{boi})}}}\,, \qquad (18)$$

where: n_2 is the exponent of the temperature curve in the dry layer, equal to 1.75; a_3 is the thermal diffusivity coefficient of the dry layer; c_3 is the specific heat capacity of the dry layer.

$$W_1 = \frac{\left(-c_1 T_0 + r_{mel}\dfrac{W_0}{100} + c_2 100\right)}{r_0}100 + W_0\,, \qquad (19)$$

where: c_1, c_2 – respectively, the specific heat capacities of the frozen and wet mixture.

The value of c_2 depends on the moisture content. As a first approximation, it can be calculated using the formula:

$$c_2 = 815 + 33.75W_0\,.$$

As seen from equation 19, all else being equal, the moisture content W_1 when casting into frozen molds is higher than when casting into wet molds.

3. Aims and Objectives of the Study

The conducted research aimed to develop a mathematical model of heat transfer in the "frozen casting mold – anti-adhesive coating – metal" system based on cellular automata. This model will allow for predicting and influencing the quality of the casting surface at the technology design stage, as well as determining the boundary conditions for the applicability of the casting method using frozen casting molds with an anti-adhesive coating for castings made of non-ferrous metals and alloys.

To achieve this goal, the following tasks were addressed:

– To model the heat transfer process in the "frozen casting mold – anti-adhesive coating – metal" system based on cellular automata.

– To verify the adequacy of the mathematical model through full-scale temperature measurements during the pouring and crystallization of AK5M2 alloy in a frozen mold with an anti-adhesive coating.

– To determine the boundary conditions for the applicability of the frozen mold casting method by modeling the crystallization of an AK5M2 alloy casting in a frozen mold.

4. Materials and Methods for Investigating Heat Transfer in the "Metal – Anti-Adhesive Coating – Frozen Mold" System

4.1. Materials and Equipment Used in the Experiment

The research was conducted using AK5M2 alloy conforming to DSTU 1583-93. The melt temperature was measured with a chromel-alumel thermocouple and an MR-64-02 millivoltmeter. The temperature of the frozen mold was measured using a TM 200 device with a *K*-type thermocouple. The alloy was melted in a CNO-6.12.4/11 resistance furnace within a ceramic crucible [7].

An anti-adhesive paint was used as the coating, with the following composition: amorphous graphite – 15…16%, crystalline graphite – 10…12%, talc – 20…25%, bentonite – 2…3%, AF-binder – 4…6%, and water to 100% and the required density. The molding mixture consisted of: sand grade 1K02b (DSTU 2138-91) – 66%, sand grade T045 (DSTU 2138-91) – 31%, clay – 3%, and water – 6…8% [8].

To prepare the molding and core mixture, the components were mixed in a muller. The components were added to the muller in the following order: the sand mixture and clay were mixed for 3-5 minutes, then 6…8% water was added, and the moistened mixture was re-mixed for another 2…3 minutes. From this molding mixture, casting molds with cores were prepared (Fig. 3).

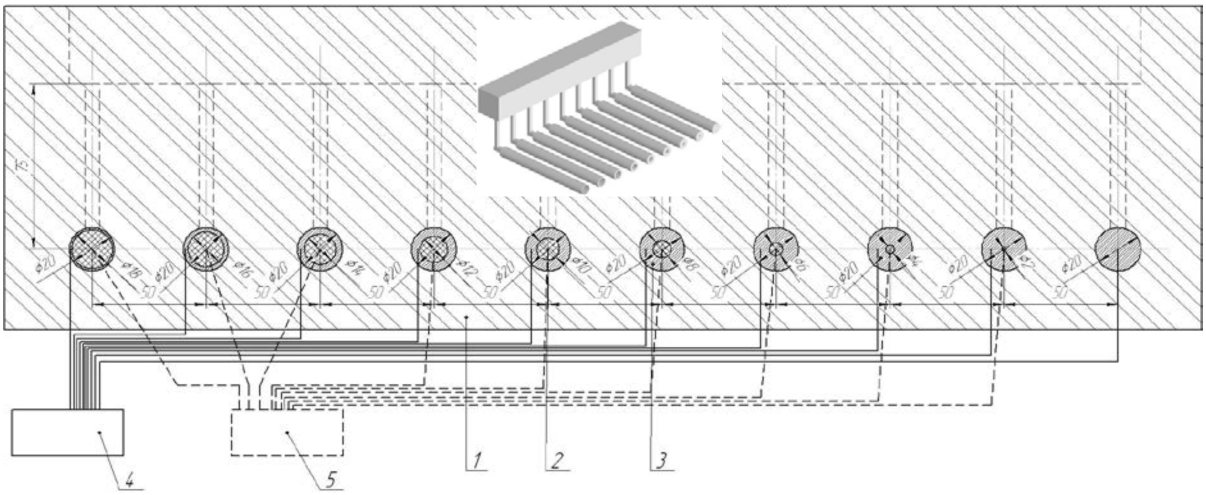

Fig. 3. Experimental Setup for Determining Melt Solidification Time in a Frozen Mold: *1* – frozen mold; *2* – frozen core; *3* – casting; *4* – recording potentiometer with thermocouples for measuring melt temperature; *5* – recording potentiometer with thermocouples for measuring temperature in the frozen core

The process of manufacturing frozen molds with a coating can be described as follows:

– The flask is placed on the pattern plate with the pattern, and the molding mixture is poured into it. The vibrator is turned on, and the mixture is compacted at a vibrator frequency of 2850 oscillations per minute and an amplitude of 10 mm for 180 seconds.

– The pattern is removed, and the anti-adhesive coating is applied to the mold cavity using a sprayer.

– After the top layer of the anti-adhesive coating dries slightly, a model cooled with liquid nitrogen, which matches the casting's configuration, is inserted into the mold cavity.

– The adjacent layer of the molding mixture freezes due to the cooled model. Once the required thickness of the frozen mixture layer is achieved, the cooled model is removed, and the resulting half-mold is sent for assembly and pouring.

4.2. Methodology for Determining the Boundary Conditions for the Applicability of Casting into Frozen Molds with an Anti-Adhesive Coating

To determine the alloy's solidification time, experimental castings were performed. A mold was manufactured, as shown in Figure 3.

The mold (*1*) was cooled to temperatures of –60 °C and –20 °C for 180 minutes. Cores (*2*), previously frozen to –60 °C and –20 °C for 60 minutes and made from the same molding material, were placed into the mold. Thermocouples (*4*) were installed in the cavity formed by the frozen mold and frozen cores. These thermocouples recorded changes in metal temperature, allowing for the determination of the melt's solidification time. Thermocouples were also placed in the center of the core (*5*) to track the temperature change dynamics within the core. Pouring was performed with AK5M2 alloy at a temperature of 700 °C.

5. Research Results

Algorithms were developed and programs were written based on the formulas described above to perform numerical calculations of the temperature fields for both the casting and the mold.

In all numerical experiments, samples with dimensions of 50×50×50 mm were modeled. At the initial time, the temperature of each automaton was –60 °C. Boundary conditions were set as follows: the upper face of the sample was heated to 700 °C, and the lower face to 20 °C. The time step was 10^{-10} seconds, and the cellular automaton size was 1 mm. The sample loading scheme is shown in Fig. 4.

Numerical experiments were conducted to simulate the propagation of the thermal front within a low-temperature casting mold. This mold was comprised of an anti-adhesive coating, dry molding mixture, wet molding mixture, and frozen molding mixture, along with zones of condensation and moisture crystallization.

To verify the program, the results obtained from the calculations were compared with those measured during the physical experiment.

The calculated curves and experimental data points for the cooling rates of AK5M2 alloy castings are presented in Fig. 5.

The comparison of the obtained calculated data and experimental data allowed us to conclude a sufficient degree of correlation for the model, which accounts for the influence of heat transfer. Thus, the mathematical model ensures reliable calculations of the non-stationary temperature fields of the casting and the wet, dry, and frozen mold, and can be used for numerical investigations. To verify the adequacy of the mathematical model, a series of computational experiments were conducted to evaluate the heat transfer coefficients at the corresponding boundaries. AK5M2 alloy was chosen as the object of study. Temperature fields of castings with a reduced wall thickness of 12 mm were considered, poured into molds with different initial temperatures of –20 °C and –60 °C.

Full-scale measurements were carried out using an analog-to-digital converter, which allowed recording thermocouple readings on a computer with a frequency of 0.1 seconds. For comparison, points were selected at the center of the casting, as well as in the mold at distances of 1, 5, and 10 mm from the boundary with the casting.

Upon contact of the liquid metal with the walls of the frozen casting mold (FCM), a sharp increase in temperatures occurs in the boundary regions (Fig. 6, curve *2*). For instance, for the –60 °C mold, this value reaches 250 °C, and for the –20 °C mold, it reaches 238 °C. The formed vapor layer causes the mold temperatures to drop to 95 °C and 190 °C, respectively. Moreover, this effect decreases or completely disappears with increasing distance from the thermal center (curves *3*, *4*). Subsequently, the temperature in the boundary region stabilizes at 97…100 °C for 70 seconds for the mold with an initial temperature of –60 °C (Fig. 6, curve *2*), and at 190 °C for 30 seconds for an FCM with an initial temperature of –20 °C (Fig. 6*a*, curve *2*).

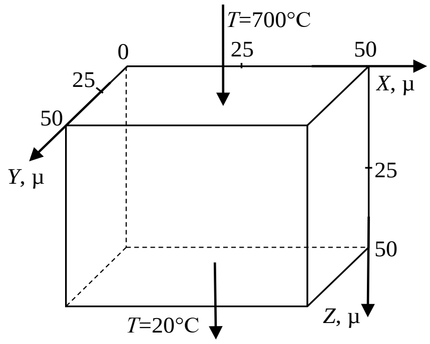

Fig. 4. Schematic of the Modeled Sample Loading

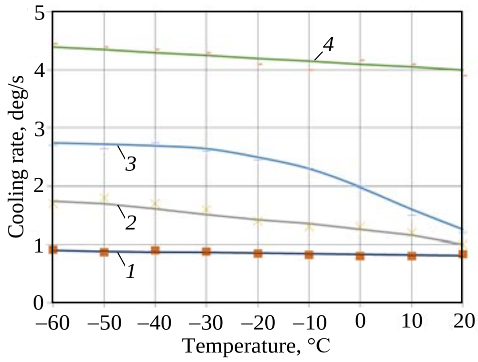

Fig. 5. Calculated dependencies of cooling rate for silumin castings on initial mold temperature: *1*, *3* – in the temperature range 720…500 °C; *2*, *4* – in the temperature range 720…577 °C; 1, 2 – reduced casting wall thickness 5 mm; *3*, *4* – reduced casting wall thickness 15 mm. Experimental data are marked with points

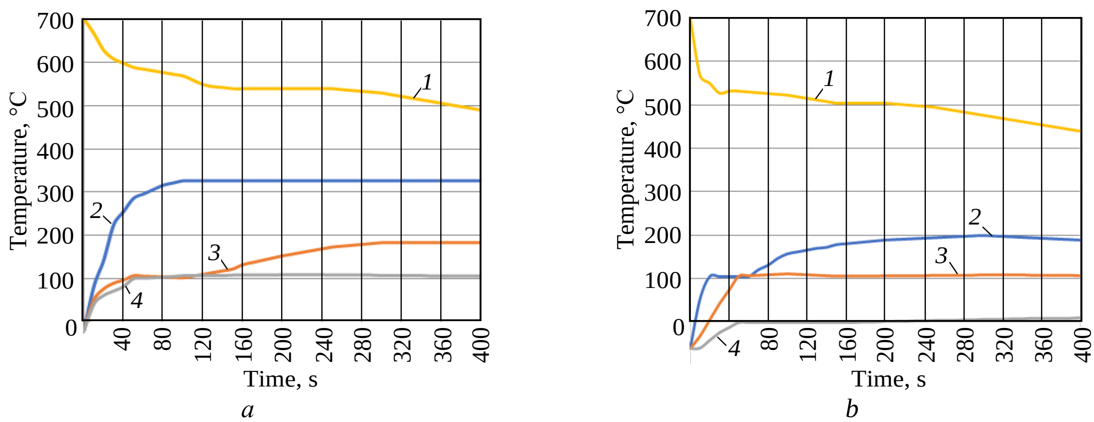

Fig. 6. Temperature Change During Cooling of AK5M2 Alloy Casting in a Low-Temperature Casting Mold with an Initial Temperature of –20 °C (*a*) and –60 °C (*b*): *1* – at the center of the casting; *2* – in the mold at a distance of 1 mm from the mold surface; *3* – 5 mm; *4* – 10 mm. Experimental data are marked with points

A practically linear increase in temperature is observed thereafter. Starting from 300 seconds, the temperature rise ceases, reaching 200 °C for the –60 °C FCM and 310 °C for the –20 °C FCM, respectively. For the FCM with an initial temperature of –20 °C (Fig. 6*a*), we observe a temperature increase to 100 °C in 110 seconds, followed by a plateau at this mark for 90 seconds, and then an increase to

175 °C. For the 10 mm layer, a plateau at 0 °C for 100 seconds is characteristic, followed by a linear temperature increase to 95 °C.

Curves *1* in the figure reflect the change in temperature at the center of the casting during its formation.

It should be noted that there is a high degree of agreement between the calculated and experimental data in the thermometry of the casting.

For comparison, the same graphs also present data from full-scale measurements, the results of which differ slightly in the initial stages. This can be explained by the fact that thermocouple readings are recorded at a lower frequency than in the calculations, and the effect of vapor layer formation is not captured. Subsequently, the readings of the full-scale and calculated values differ insignificantly due to the slowing of the temperature increase rate in the mold and the decrease in temperature in the metal.Based on the conducted calculations, confirmed by experimental data, a nomogram was constructed to determine the boundary conditions for applying the casting method into frozen molds with frozen cores (Fig. 7). A mathematical dependence was also derived for the depth of mold (core) heating δ as a function of casting thickness *x* [9].

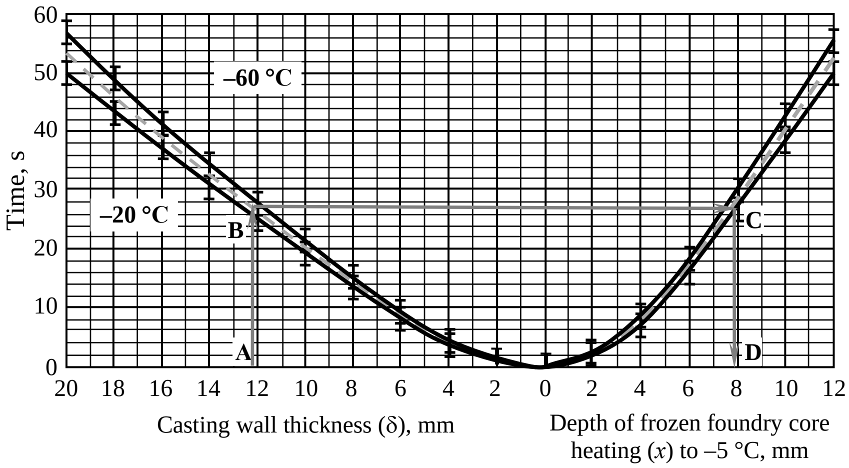

Fig. 7. Nomogram for Determining the Thickness of a Frozen Core Depending on the Casting Wall Thickness

This nomogram allows for determining the minimum thickness of a frozen core depending on the casting wall thickness and the cooling temperature of the mold (core). One selects the casting wall thickness (point A), draws a vertical line to intersect with the curve. The curve is chosen depending on the temperature of the frozen mold: –60 °C or –20 °C (point B). Then, a horizontal line is drawn to intersect with the curve representing the depth of heating of the frozen core (point C). Finally, a vertical line is dropped to intersect with the axis, determining the minimum core thickness (point D) [10].

A mathematical dependence was obtained for the depth of heating (δ) of the mold (core) to –5.0 °C, at which the strength of the frozen mold (core) becomes comparable to that of a sand-clay mold, as a function of the casting wall thickness (*x*).

$$\delta = -0.002x^3 + 0.065x^2 + 0.206x + 0.110.$$

6. Conclusions

1. Development of a mathematical model for heat transfer in the “frozen casting mold – anti-adhesive coating – metal” system based on cellular automata.

In this work, a mathematical model was successfully developed that describes the complex heat transfer processes occurring in a system including a frozen casting mold, an anti-adhesive coating, and molten metal. The model is based on the stochastic excitable cellular automata (SECA) method, which allows for accounting for the discrete structure of materials and changes in their states over time. This is particularly important when modeling phenomena such as phase transitions of water in the mold (ice melting, water evaporation, steam condensation) and their impact on thermal processes. The developed model considers the thermophysical properties of materials (temperature, thermal conductivity, heat capacity, and other parameters), ensuring its adequacy for analyzing thermal phenomena in the system under consideration.

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2. Conducting heat transfer calculations in the "frozen casting mold – anti-adhesive coating – metal" system based on the developed mathematical model.

Based on the created mathematical model, a comprehensive set of calculations was performed, aimed at quantitatively evaluating the thermal processes in the "frozen casting mold – anti-adhesive coating – metal" system. The calculations provided data on the spatial and temporal distribution of temperatures, casting cooling rates, metal solidification kinetics, and the dynamics of temperature changes in various mold zones. The obtained results can be used to optimize the technological parameters of the casting process, such as pouring temperature, mold temperature, and anti-adhesive coating thickness, which, in turn, contributes to improving casting quality and reducing defects.

3. Experimental confirmation of the adequacy of the mathematical model.

An important stage of the work was the experimental verification of the adequacy of the developed mathematical model. For this purpose, full-scale experiments were conducted using real casting molds, anti-adhesive coatings, and metal alloys. A comparison of the calculation results with experimental data showed a sufficiently high degree of their agreement, which confirms the reliability of the mathematical model and the possibility of its application for practical purposes. In particular, the model adequately describes the influence of the initial mold temperature on the casting cooling rate and temperature changes in the mold's boundary regions. This allows the model to be used for predicting the thermal conditions of the casting process and selecting optimal parameters that ensure the production of castings with desired properties.

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