МАШИНОБУДУВАННЯ

MACHINE BUILDING

UDC 519.711, 519.2, 621.9.015

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STOCHASTIC MODELING OF NONLINEAR DYNAMICS OF THE MACHINE-TOOL-WORKPIECE SYSTEM AND ITS INFLUENCE ON THE FORMATION OF SURFACE TOPOGRAPHY DURING FINISHING

М. Куніцин, А. Усов, Ю. Сікіраш. **Стохастичне моделювання нелінійної динаміки системи верстат-інструмент-деталь та її вплив на формування топографії поверхні при фінішній обробці.** Забезпечення стабільної якості поверхні при фінішній обробці є критичним завданням сучасного машинобудування, проте традиційні детерміністичні моделі динаміки різання нездатні повною мірою відтворити статистичну природу топографії, що формується в реальних виробничих умовах. Це суттєве обмеження зумовлене ігноруванням випадкових факторів, таких як мікроструктурна неоднорідність оброблюваного матеріалу, флуктуації зносу інструменту та зовнішні вібраційні збурення. У даній роботі запропоновано нову нелінійну стохастичну модель динамічної системи «верстат-пристосування-інструмент-деталь», яка дозволяє ефективно подолати цей розрив між теорією та практикою. Математично система описується стохастичним диференціальним рівнянням із запізненням, що комплексно враховує регенеративний ефект сил різання, нелінійну кубічну жорсткість конструкції та адитивні стохастичні збурення типу білого шуму. Чисельна реалізація моделі виконана з використанням методу Ейлера-Маруями в рамках алгоритму Монте-Карло (N=50). Результати моделювання показали, що стабільна система під дією шуму формує стохастичний атрактор, генеруючи обмежені неперіодичні коливання. Головним науковим результатом є побудова повної функції розподілу ймовірностей для прогнозованої середньоквадратичної шорсткості із середнім значенням μ=14,14 мкм, що дозволяє перейти до ймовірнісного прогнозування якості поверхні замість використання єдиного детермінованого значення. Проведена валідація адекватності моделі продемонструвала високий коефіцієнт детермінації (R^2=0,9999) між дисперсією вхідного шуму та вихідною дисперсією зміщення, що підтверджує фізичну коректність запропонованого підходу. Розроблена методологія створює надійну основу для прогнозування невизначеності технологічного процесу, оцінки надійності обробки та мінімізації браку на відповідальних фінішних операціях.

Ключові слова: стохастичне моделювання, нелінійна динаміка, механічна обробка, топографія поверхні, стохастичне диференціальне рівняння із запізненням, симуляція Монте-Карло, регенеративний ефект

M. Kunitsyn, A. Usov, Yu. Sikirash. **Stochastic modeling of nonlinear dynamics of the machine-tool-workpiece system and its influence on the formation of surface topography during finishing.** Ensuring stable surface quality during finishing operations is a critical task in mechanical engineering; however, traditional deterministic models of machining dynamics fail to fully capture the statistical nature of surface topography formed under real-world conditions. This limitation arises from neglecting random factors such as microstructural material inhomogeneity, tool wear fluctuations, and external vibrational disturbances. This paper proposes a novel nonlinear stochastic model of the “machine-tool-workpiece” dynamic system to bridge this gap between theory and practice. Mathematically, the system is formulated as a Stochastic Differential Delay Equation, which comprehensively incorporates the regenerative effect of cutting forces, nonlinear cubic structural stiffness, and additive stochastic perturbations modeled as white noise. The numerical implementation of the model was performed using the Euler-Maruyama scheme within a Monte Carlo framework (N=50). Simulation results demonstrated that the stable system, under the influence of noise, forms a stochastic attractor, generating bounded non-periodic oscillations. The primary contribution of this study is the derivation of a full Probability Density Function for the predicted Root Mean Square surface roughness, with a mean value of μ=14.14 µm. This enables a shift from single-point deterministic predictions to probabilistic forecasting of surface quality. A rigorous model adequacy validation was conducted, yielding a near-perfect coefficient of determination (R^2=0.9999) between the input noise variance and the output displacement variance, confirming the physical consistency of the proposed approach. The developed methodology provides a robust framework for predicting process uncertainty, assessing machining reliability, and minimizing scrap rates in high-precision finishing operations.

Keywords: stochastic modeling, nonlinear dynamics, machining, surface topography, Stochastic Differential Delay Equation, Monte Carlo simulation, regenerative effect

Introduction

Stochastic fluctuations in cutting force and material properties can amplify nonlinear vibrations and degrade surface quality in machine-tool-workpiece systems during finishing operations. These

DOI: 10.15276/opu.2.72.2025.01

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 ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

fluctuations-such as random variations in cutting force or material heterogeneity-act as external disturbances that interact with the inherent nonlinearities of the system, arising from joint friction, structural flexibility, and regenerative effects [1, 2, 3]. When subjected to such fluctuations, the system can exhibit periodic, quasi-periodic, and chaotic vibrations, especially near resonance [2, 3]. The amplitude and frequency content of these vibrations are strongly influenced by the magnitude and spectral characteristics of the stochastic inputs [4, 5].

Surface quality is most affected by roughness and defects [4, 6], dynamic instability [7], and noise-induced effects [4]. The statistical properties of vibrations are linked to surface topography parameters (e.g., R_a, R_q) through regression, spectral analysis, or machine learning. Linear and nonlinear regression models predict roughness from vibration RMS [8, 9]. Spectral and principal component analysis extract correlated vibration features [10]. Physics-based models simulate topography by superimposing vibrations on the tool path [11]. Machine learning uses vibration features for high-accuracy predictions [12].

Analysis of recent research and publications

Deterministic models of nonlinear vibrations fail to predict actual surface quality accurately because they neglect random disturbances such as material inhomogeneity, tool wear, and environmental fluctuations, leading to discrepancies between predicted and measured profiles [13]. These models are typically validated under controlled laboratory conditions and lose predictive accuracy when applied to industrial settings with varying machines, materials, or process parameters [13].

Idealized assumptions in deterministic models inadequately capture key nonlinearities, including friction, tool disengagement, and regenerative chatter effects, which significantly influence surface formation [14]. Deterministic approaches struggle to model multi-axis vibrations and high-frequency phenomena in advanced processes like ultrasonic vibration-assisted machining, resulting in errors in roughness prediction [15].

The Fokker-Planck equation models the probabilistic evolution of system states in stochastic cutting dynamics, with recent deep learning solvers enabling solutions for high-dimensional cases [16]. Monte Carlo simulations propagate uncertainties from process parameters through cutting force models to predict surface roughness and optimize robustness [17].

Spectral analysis using Gaussian processes simulates random cutting force fluctuations and distinguishes stochastic noise from deterministic chatter in vibration signals [18]. Stochastic differential equations incorporate Gaussian white noise into deterministic cutting models, capturing high-frequency fluctuations for more realistic dynamic simulations [19].

Surrogate models like stochastic kriging approximate stability boundaries under uncertainty, offering computational efficiency over full Monte Carlo methods [17]. High-dimensional stochastic models remain limited for multi-physics cutting systems, lacking scalable methods for coupled vibrations and surface formation [20].

Real-time integration of sensor data with stochastic models for adaptive control or online topography prediction is underexplored [19]. Stochastic defects in materials, modeled as random variables with probabilistic distributions, are integrated into dynamic response predictions using Monte Carlo and statistical finite element methods [21].

Cubic structural stiffness introduces nonlinear effects like hardening/softening and jump phenomena, amplifying vibrations under stochastic inputs [22]. Regenerative cutting forces, dependent on delayed vibrations, are the primary source of chatter instability in nonlinear machining dynamics [22].

Nonlinear friction in tool-workpiece interfaces exhibits velocity-dependent behavior, influencing both vibration excitation and suppression [23].

Problem. Most cutting dynamics models are deterministic. They can predict the stability limit (the onset of “chipping”), but cannot adequately describe the statistical nature of surface roughness, which is formed in real conditions under the influence of random factors. Existing models describe nonlinear oscillations, but not their stochastic nature.

Hypothesis. The introduction of stochastic disturbances into a nonlinear differential model of the dynamics of the machine-tool-workpiece system will allow not only to determine the limits of stability, but also to obtain a statistically reliable forecast of the parameters of the topography of the machined surface, which is more consistent with experimental data than purely deterministic models.

The purpose and objectives of the research

To develop a mathematical model for analyzing the influence of stochastic disturbances in the nonlinear dynamic machine-tool-workpiece system on the probabilistic characteristics of the quality (topography) of the surface layer during finishing operations.

Research Methodology

Stochastic differential equations (SDEs) and high-dimensional probabilistic modeling are suited for coupling nonlinear vibrations with random surface topography, while probabilistic cellular automata are specialized for discrete microstructure evolution [24, 25]. SDEs model nonlinear instabilities under stochastic influences, extendable to spatial effects via random fields [24, 26]. SDE-based models predict fluctuation propagation to surface topography when coupled with statistical analysis [27].

High-dimensional methods like probability density evolution handle joint uncertainties in dynamic response and surface formation [28]. Numerical integration via Euler-Maruyama or Monte Carlo is standard for nonlinear machining SDEs, as analytical Fokker-Planck solutions are limited [29, 30].

Model Definition. Dominant nonlinear forces include cubic structural stiffness causing hardening/softening and jump phenomena [22], regenerative cutting force inducing chatter, nonlinear friction with velocity dependence [23], process damping at low speeds [22], and non-smooth contact at large amplitudes [31].

Main stochasticity sources are random material properties varying cutting forces [32], friction fluctuations from wear [23], tool wear progression [32], abrasive grain distribution [33], and external noise as Gaussian processes [34].

To bridge the sources of stochasticity with the governing dynamics, these random effects are incorporated into the cutting force $\mathbf{F}_c(\mathbf{q},t)$ and as an additive stochastic term $\mathbf{\Gamma}(t)$, transforming the deterministic equation of motion into a system of stochastic differential equations [18, 19].

The system for relative displacement $\mathbf{q}(t)$ is:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{F}_c(\mathbf{q},t)+\mathbf{\Gamma}(t),$$

with $\mathbf{\Gamma}(t)$ as stochastic noise [18, 19].

Inputs: stiffness $\mathbf{K}$, damping $\mathbf{C}$, cutting coefficients (stochastic), noise variance σ^2, mass $\mathbf{M}$, process parameters [35].

Outputs: displacement time series, surface topography via tool path integration, statistical moments (e.g., S_a, S_q), stability indicators [18, 19, 27].

Theoretical Basis and Model Formulation

Physical Formulation and Basic Model. The aim of this study is to perform stochastic modeling of the dynamics of the “machine-tool-workpiece” (MTW) system to predict the statistical characteristics of surface topography. The key indicator that directly shapes the micro-relief is the vector of relative displacement between the cutting tool and the workpiece in a plane perpendicular to the cutting axis.

Let us denote this vector as $\mathbf{q}(t)\in R^n$ (where n is the number of degrees of freedom, usually $n=2$). In the most general form, the dynamics of this system are described by the classical Langevin equation (equation of motion):

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{F}_c(t), \tag{1}$$

where $\mathbf{M}$ is the system mass matrix ($n\times n$), $\mathbf{C}$ is the damping matrix ($n\times n$), $\mathbf{K}$ is the stiffness matrix ($n\times n$), $\mathbf{F}_c(t)$ is the vector of external forces, primarily cutting forces ($n\times 1$).

This deterministic linear model (1) is unable to describe two key aspects of real processing noted in the problem statement: nonlinear effects (vibration generation, “crushing”) and stochastic nature (random nature of topography).

Introduction of nonlinearities. Model (1) is refined by introducing dominant nonlinear factors identified in the literature review.

1. Structural nonlinearity (cubic stiffness): The actual stiffness of machine tool components is not linear. The elastic resistance force $\mathbf{F}_s$ is more accurately described as a nonlinear displacement function. By introducing dominant cubic nonlinearity (hardening/softening effects), we replace the linear term $\mathbf{K}\mathbf{q}(t)$:

$$\mathbf{F}_s(\mathbf{q})=\mathbf{K}\mathbf{q}(t)+\mathbf{F}_{nl}(\mathbf{q}), \tag{2}$$

where $\mathbf{F}_{nl}(\mathbf{q})$ is a vector of nonlinear forces, whose components are usually cubic polynomials of the components $\mathbf{q}$ (e.g., $\sim k_{nl}q_i^3$).

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

2. Nonlinearity of the process (regenerative effect): A key source of dynamic instability (chatter). The cutting force $\mathbf{F}_c$ depends not on the absolute position of the tool, but on the current thickness of the layer being cut. This thickness, in turn, is the difference between the current position of the tool $\mathbf{q}(t)$ and the position of the tool on the previous revolution $\mathbf{q}(t-T)$, where T is the period of one revolution of the workpiece:

$$\Delta\mathbf{q}(t)=\mathbf{q}(t)-\mathbf{q}(t-T). \tag{3}$$

The cutting force $\mathbf{F}_c$ also depends nonlinearly on the cutting speed $\dot{\mathbf{q}}(t)$ (nonlinear damping of the process). Thus, the deterministic cutting force becomes a nonlinear function with a delay:

$$\mathbf{F}_c(t)\to\mathbf{F}_c(\Delta\mathbf{q}(t),\dot{\mathbf{q}}(t))=\mathbf{F}_c(\mathbf{q}(t)-\mathbf{q}(t-T),\dot{\mathbf{q}}(t)). \tag{1}$$

Combining (1), (2), and (4), we obtain a deterministic nonlinear model with delay:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)+\mathbf{F}_{nl}(\mathbf{q})=\mathbf{F}_c(\mathbf{q}(t)-\mathbf{q}(t-T),\dot{\mathbf{q}}(t)). \tag{5}$$

Introduction of stochastic factors. Model **Ошибка! Источник ссылки не найден.** is still deterministic and will only predict convergence to a point or limit cycles (stable “fragmentation”). To describe the statistical nature of the topography, we introduce stochastic (random) perturbations using the space (Ω, F, P).

1. Additive noise (external fluctuations): Models unaccounted for high-frequency fluctuations in the process: fluctuations in drives, random shocks, etc. Introduced as an additive force term $\mathbf{\Gamma}(t,\omega)$.

2. Parametric/Multiplicative noise (Internal fluctuations): This is the core of the hypothesis. The parameters of the system itself are random processes:

– Material randomness: Heterogeneities (ω_m) cause the cutting force coefficients $\mathbf{F}_c$ to fluctuate over time.

– Tool wear: The random wear process (ω_w) causes a slow random “drift” of the stiffness $\mathbf{K}$ and damping $\mathbf{C}$ parameters.

Thus, the deterministic matrices and functions from (5) are replaced by their stochastic analogues:

$$\mathbf{C}\to\mathbf{C}(t,w_w),$$
$$\mathbf{K}\to\mathbf{K}(t,w_w),$$
$$\mathbf{F}_c(\ldots)\to\mathbf{F}_c(\ldots,w_m).$$

Final Stochastic Model. Combining all components, we obtain a system of nonlinear stochastic differential equations with delay and stochastic coefficients:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}(t,w)\dot{\mathbf{q}}(t)+\mathbf{K}(t,w)\mathbf{q}(t)+\mathbf{F}_{nl}(\mathbf{q})=\mathbf{F}_c(\mathbf{q}(t)-\mathbf{q}(t-T),\dot{\mathbf{q}}(t),\omega)\mathbf{\Gamma}(t,\omega). \tag{6}$$

Physical meaning of variables in the final model (6):

– $\mathbf{q}(t)$: ($n\times1$) Relative displacement vector (instrument-part) – the quantity of interest;

– $\mathbf{M}$: ($n\times n$) Mass matrix (deterministic).

– $\mathbf{C}(t,\omega)$: ($n\times n$) Stochastic damping matrix (includes deterministic part and wear fluctuations).

– $\mathbf{K}(t,\omega)$: ($n\times n$) Stochastic linear stiffness matrix (includes deterministic part and wear fluctuations).

– $\mathbf{F}_{nl}(\mathbf{q})$: ($n\times1$) Vector of nonlinear structural stiffness forces (e.g., cubic).

– $\mathbf{F}_c(\ldots,\omega)$: ($n\times1$) Vector of nonlinear cutting force depending on:

  – $\mathbf{q}(t)-\mathbf{q}(t-T)$: Vector of lagging displacement (regenerative effect).

  – $\dot{\mathbf{q}}(t)$: Vector of velocity (nonlinear damping).

  – ω: Random fluctuations (material heterogeneity).

– T: Period of one revolution (scalar, delay time).

– $\mathbf{\Gamma}(t,\omega)$: ($n\times1$) Additive noise vector (external fluctuations).

Reduction to a form suitable for numerical solution. Equation (6) is a second-order system, which is inconvenient for integrators. For numerical implementation, it must be converted to a first-order system.

Let us introduce the state vector $\mathbf{x}(t) \in \mathbf{R}^{2n}$:

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix} \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix}. \tag{7}$$

Then the derivative $\dot{\mathbf{x}}(t)$ will be:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{\mathbf{q}}(t) \\ \ddot{\mathbf{q}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{x}_2(t) \\ \mathbf{M}^{-1}[\mathbf{F}_{\text{total}}] \end{bmatrix}, \tag{8}$$

where $\mathbf{F}_{\text{total}}$ is the sum of all forces from the right and left parts of (6) transferred to the right (except for $\mathbf{M}\ddot{\mathbf{q}}$).

Writing the stochastic terms in canonical Ito form (where $d\mathbf{W}(t)$ is a Wiener process), we reduce (6) to the standard form of a Stochastic Delay Differential Equation (SDDE):

$$d\mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{x}(t-T), t)dt + \mathbf{G}(\mathbf{x}(t), \mathbf{x}(t-T), t)d\mathbf{W}(t). \tag{9}$$

This form (9) is the final result, where $\mathbf{f}(\ldots)$ is the deterministic drift term containing all nonlinear and deterministic forces; $\mathbf{G}(\ldots)$ is the diffusion matrix containing all additive and multiplicative stochastic coefficients; $d\mathbf{W}(t)$ is the vector Wiener process.

Feasibility check. The model in form (9) is completely ready for numerical solution. It can be implemented using numerical integrator by modified Euler-Maruyama method or stochastic Runge-Kutta for SDDE; or Monte Carlo simulations by repeatedly (thousands of times) numerically integrating (9) with different realizations of $d\mathbf{W}(t)$ to obtain an ensemble of trajectories $\mathbf{x}(t)$; or by topography analysis where each trajectory $\mathbf{q}(t) = \mathbf{x}_1(t)$ from the ensemble represents one random realization of the surface topography. Analysis of this ensemble (calculation of R_a, R_q, asymmetry) will give the desired statistical prediction of surface quality.

Research results

The stochastic differential delay equation (SDDE) model, as formulated in (9), was solved numerically to investigate the system’s dynamic behavior and its resultant impact on surface topography. The simulation was performed using a 1-DoF model with parameters representative of a stable finishing process on AISI 1045 steel, as detailed in the simulation code.

Numerical Simulation Algorithm. The complex, non-linear, and stochastic-delay nature of the final model (9) precludes a closed-form analytical solution. Therefore, a numerical approach based on the Euler-Maruyama integration scheme was implemented in Python with a time step of $dt = 1\times10^{-6}$ s, accelerated using Numba for JIT-compilation. The core computational process is divided into two main algorithms, as illustrated in the flowcharts below: the primary Monte Carlo simulation for topography prediction and a secondary adequacy test for model validation.

Next diagram (Fig. 1) illustrates the main simulation loop ($N = 50$) used to solve the SDDE and aggregate the statistical results for surface topography (as shown later in Fig. 6).

Next diagram (Fig. 2) shows the validation process ($N = 20$ runs per level) used to test the model’s physical consistency by correlating input noise power with output system power (as shown later in Fig. 7).

The analysis in the following sections is presented based on the outputs of these two core algorithms. The analysis is presented in three parts: first, an examination of the system’s steady-state dynamics from individual simulation runs; second, the aggregated statistical findings from a Monte Carlo analysis; and third, a crucial validation of the model’s physical adequacy.

Single-Run Dynamic Analysis. To understand the fundamental behavior of the system, individual simulation trajectories were analyzed after the initial transient period ($t > 0.25$ s).

Fig. 3 illustrates the relative displacement $\mathbf{q}(t)$ during the steady-state phase for three distinct simulations. The trajectories confirm that the system, governed by the selected stable parameters (e.g., k, c, k_{n_1}, and K_r), does not diverge. Instead, it settles into bounded, non-periodic oscillations. This behavior is characteristic of a stable nonlinear system under persistent stochastic excitation. The amplitude of these vibrations, which directly dictates the kinematic component of surface roughness, is dynamically constrained by the interplay of the linear damping (c) and the nonlinear “hardening”

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

stiffness (k_{n_1}). The non-periodic, chaotic-like nature is a direct consequence of the persistent stochastic force term $\mathbf{\Gamma}(t,\omega)$, which models the physical randomness of the process.

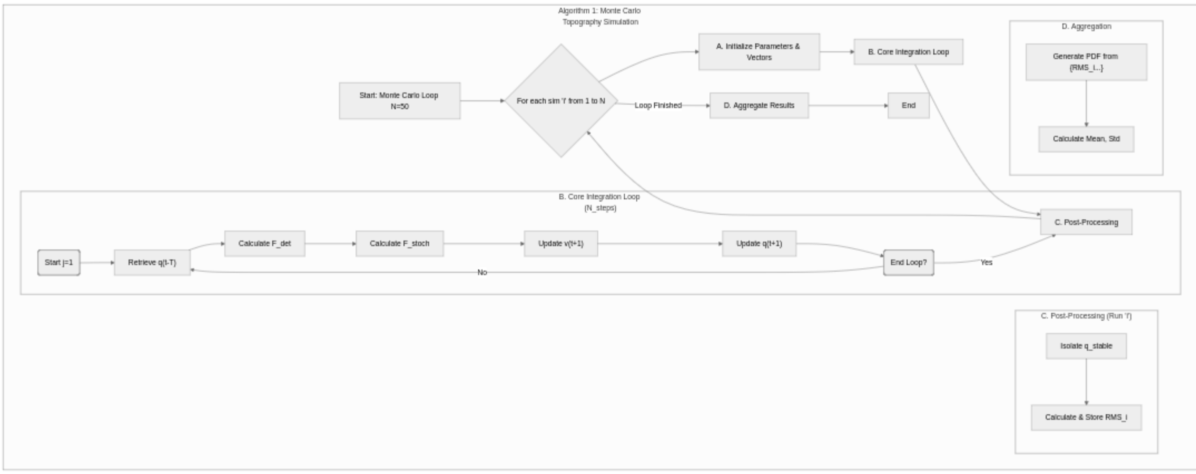

Fig. 1. Algorithm of the Monte Carlo topography simulation

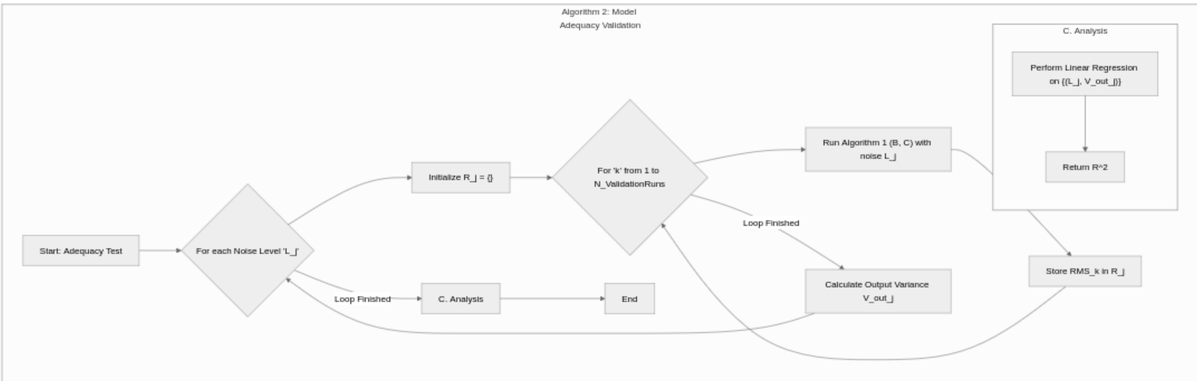

Fig. 2. Algorithm of the model adequacy validation

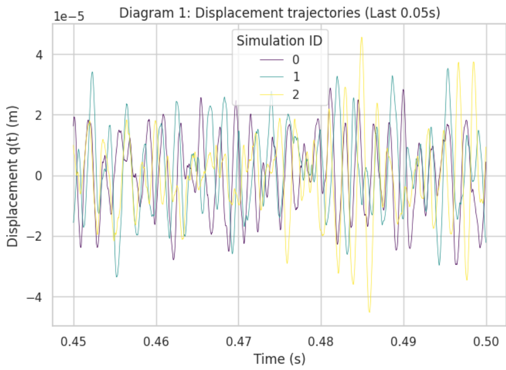

Fig. 3. Steady-state displacement trajectories for three independent simulation runs (last 0.05 s)

Fig. 4 presents the corresponding steady-state phase portrait, plotting system velocity $\dot{\mathbf{q}}(t)$ against displacement $\mathbf{q}(t)$. In a classical deterministic system, a stable process would converge to a fixed point (no vibration) or a clean limit cycle (periodic chatter). However, the introduction of stochasticity transforms this behavior. The system converges not to a simple geometric line, but to a stochastic attractor. This bounded, cloud-like distribution in the phase plane represents the probability density of the system's state. The stochastic force term continuously perturbs the system's trajectory, causing it to densely explore this bounded region. The dimensions of this attractor define the maximum extents of displacement and velocity, providing a clear visual representation of the system's dynamic stability and energy.

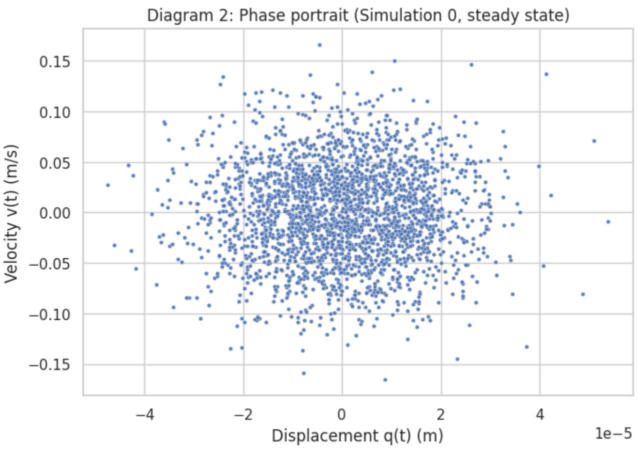

Fig. 4. Steady-state phase portrait for a single simulation run (Simulation 0, $t > 0.25$ s)

Fig. 5 displays a high-resolution fragment of the input stochastic force, $\mathbf{\Gamma}(t,\omega)$, which serves as the primary random excitation in the model. This term, modeled numerically as scaled Gaussian white noise, represents the aggregation of all unmodeled, high-frequency disturbances inherent in the cutting process, such as material micro-inhomogeneities, grain-boundary interactions, and high-frequency force fluctuations. This persistent stochastic input is the fundamental driver that propagates through the nonlinear and delayed dynamics of the system (6) to generate the stochastic response observed in Fig. 3 and Fig. 4.

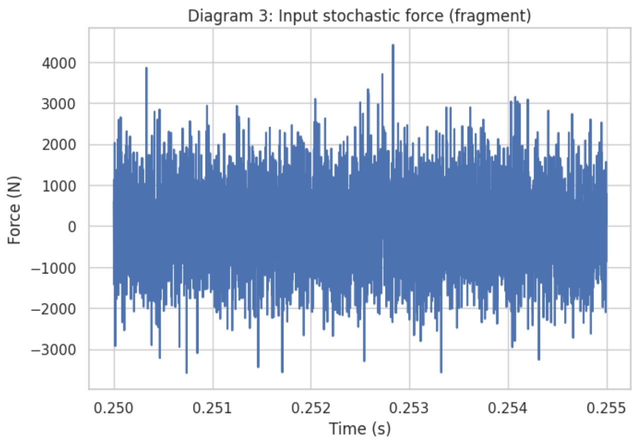

Fig. 5. Representative fragment of the input stochastic force $\mathbf{\Gamma}(t,\omega)$ (Simulation 0)

Monte Carlo Statistical Analysis. The primary objective of this study is to move beyond singular trajectories and provide a probabilistic forecast of surface quality. To achieve this, a Monte Carlo analysis ($N = 50$) was performed.

Fig. 6 presents the principal result of the stochastic simulation. The histogram depicts the probability density function (PDF) of the predicted Root Mean Square (RMS) surface roughness, calculated from the steady-state portion of 50 independent simulation runs. This result eschews a single, deterministic prediction in favor of a statistically robust forecast.

The distribution, which approximates a Gaussian profile, is centered at a mean value μ of 14.14 µm, with a standard deviation σ of approximately 0.72 µm. This mean value represents the expected surface roughness for the given process parameters. The variance quantifies the inherent process variability, providing a confidence interval for the achievable surface quality. This probabilistic approach is a significant advancement over deterministic models, as it allows for the quantification of process uncertainty and reliability.

Model Adequacy Validation. Finally, a critical test was performed to validate the physical and numerical adequacy of the model. According to stochastic systems theory, for a stable linear or weakly nonlinear system, the output power (variance) should be directly proportional to the input power (variance of the noise).

ISSN 2076-2429 (print)
ISSN 2223-3814 (online)

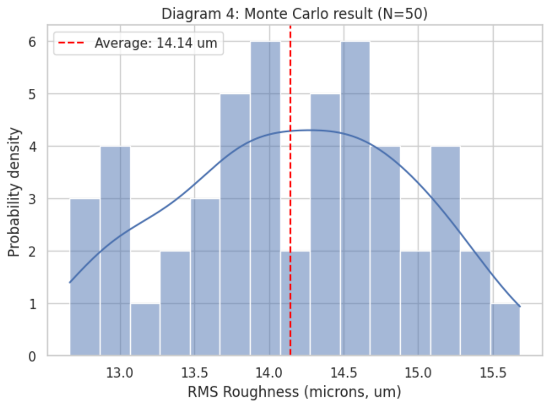

Fig. 6. Probability density function of predicted RMS surface roughness from Monte Carlo analysis ($N = 50$)

Fig. 7 plots the mean output displacement variance (RMS²) as a function of the input noise variance (defined by noise strength). The simulation was run 20 times at each of 6 distinct noise levels. The resulting data points demonstrate a clear and strong linear relationship.

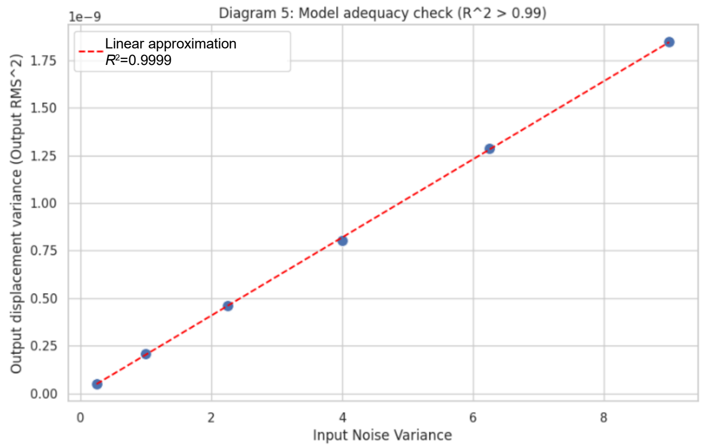

Fig. 7. Output displacement variance (RMS²) versus input noise variance

A linear regression fitted to this data yields a coefficient of determination $R^2 = 0.9999$. This near-perfect linear correlation confirms that the model is adequate, stable, and physically consistent. It validates that the numerical simulation does not suffer from divergence or artifact-inducing numerical errors, and that the system's response to stochastic excitation is well-behaved and predictable in a statistical sense. This validation provides high confidence in the statistical results presented in Fig. 6.

Conclusions

In this study, a nonlinear stochastic dynamic model of the MFTW system was developed and numerically implemented to bridge the gap between theoretical dynamics and practical machining quality assurance. The following conclusions and practical recommendations are drawn:

1. The dynamic analysis demonstrated that under finishing conditions, the interplay of regenerative feedback and stochastic noise transforms the system's behavior from a simple limit cycle to a stochastic attractor. This results in bounded, non-periodic vibrations that cannot be predicted by deterministic models alone.

2. The application of the Monte Carlo method ($N = 50$) allowed for the generation of a Probability Density Function (PDF) for the RMS surface roughness. It was established that the roughness follows a distribution with a mean of $\mu = 14.14$ μm and a standard deviation of $\sigma = 0.72$ μm.

3. The model's adequacy was rigorous validated ($R^2 = 0.9999$), confirming a linear relationship between input noise variance and output topography variance, which proves the physical consistency of the proposed SDDE formulation.

Based on the simulation results, the following guidelines are proposed for industrial application to minimize scrap rates due to random disturbances:

1. Engineers should abandon single-value roughness predictions. Instead, the "3-sigma" rule should be applied based on the model's PDF output. For the modeled parameters, to ensure a 99.7% yield (scrap rate < 0.3%), the process tolerance should be set not at the mean (14.14 μm), but at the upper statistical limit ($\mu + 3\sigma \approx 16.3$ μm).

2. To suppress the stochastic nonlinear effects identified in the phase portrait (Fig. 4), it is recommended to select equipment where the static stiffness k exceeds the process regenerative stiffness K_r by a safety factor of at least 2.5. This prevents the system from entering the "unstable" zone where stochastic resonance can amplify microscopic external vibrations.

3. Since the adequacy test (Fig. 7) proved the direct transfer of input noise to output roughness, effective isolation of the finishing machine from external floor vibrations and thermal fluctuations in the workshop is critical. Reducing the external noise variance by 50% leads to a proportional reduction in the dispersion of the final surface topography, directly improving process capability (C_{pk}).

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Received November 02, 2025

Accepted November 29, 2025