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V. Kurhan
Odessa Polytechnic National University, Shevchenko Ave. 1, Odesa, Ukraine, 65044, e-mail: 10127013@stud.op.edu.ua

ANALYSIS OF BUCKET ELEVATOR DRIVE MOUNTING STRUCTURES BASED ON GRAPH MODELS

В. Курган. **Аналіз конструкцій кріплення приводу ковшових елеваторів на основі графових моделей.** У статті наведено результати аналізу та синтезу приводів ковшових елеваторів на основі комбінованих графових моделей у механіці. Дослідження виконано з використанням теорії модифікованих кінематичних графів, що дозволяє подати приводи ковшових елеваторів у вигляді пружних механічних систем з механічним зворотним зв'язком. Запропонований підхід забезпечує формалізований аналіз структур існуючих приводів і створює підґрунтя для синтезу нових конструктивних рішень з розширеними функціональними можливостями. Проведено огляд та систематизацію основних типів графових моделей, що застосовуються для розв'язання задач механіки, зокрема кінематичних, силових та комбінованих графів. Показано доцільність використання модифікованих кінематичних графів для аналізу керованих пружних систем, що характеризуються негативною або нульовою рухомістю. Для оцінювання можливостей керування пружними характеристиками механізмів використано показники ступеня рухомості та цикломатичного числа. У роботі запропоновано критерій вибору оптимального конструктивного рішення на основі енергії модифікованого кінематичного графа, яка визначається з використанням спектральної теорії графів. Матриці суміжності формуються з урахуванням вагових коефіцієнтів, що відображають конструктивно-технологічні фактори та напрям дії пружних сил. На прикладі двох структур кріплення приводів ковшових елеваторів виконано порівняльний аналіз, який дозволив обґрунтувати вибір більш раціональної структури за критерієм мінімальної енергії графа. Отримані результати підтверджують ефективність застосування модифікованих кінематичних графів для формалізації та алгоритмізації процесів структурного аналізу й синтезу приводів ковшових елеваторів з урахуванням конструктивних і технологічних обмежень.
Ключові слова: ковшовий елеватор, привід, пружна система, модифікований кінематичний граф, механічний зворотний зв'язок

V. Kurhan. **Analysis of Bucket Elevator Drive Mounting Structures Based on Graph Models.** The paper presents the results of the analysis and synthesis of bucket elevator drives based on combined graph models in mechanics. The study is carried out using the theory of modified kinematic graphs, which makes it possible to represent bucket elevator drives as elastic mechanical systems with mechanical feedback. The proposed approach provides a formalized analysis of the structures of existing drives and creates a basis for the synthesis of new design solutions with extended functional capabilities. A review and systematization of the main types of graph models used to solve problems in mechanics is performed, including kinematic, force, and combined graphs. The expediency of using modified kinematic graphs for the analysis of controlled elastic systems characterized by negative or zero mobility is demonstrated. The degree of mobility and the cyclomatic number are used to assess the controllability of elastic characteristics of mechanisms. The paper proposes a criterion for selecting an optimal design solution based on the energy of a modified kinematic graph, which is determined using spectral graph theory. Adjacency matrices are formed taking into account weighting coefficients that reflect design and technological factors as well as the direction of elastic force action. Using the example of two bucket elevator drive mounting structures, a comparative analysis is performed, which substantiates the selection of a more rational structure according to the criterion of minimum graph energy. The obtained results confirm the effectiveness of using modified kinematic graphs for the formalization and algorithmization of structural analysis and synthesis processes of bucket elevator drives, taking into account design and technological constraints.
Keywords: bucket elevator, drive, elastic system, modified kinematic graph, mechanical feedback

1. Introduction

The development of the agro-industrial complex necessitates improving the efficiency of technological processes for transporting bulk materials. A significant role in these processes is played by bucket elevators, which are widely used in grain processing and related industries. Of particular interest are self-supporting bucket elevators, whose design features impose limitations on the mass and overall dimensions of the drive unit, thereby complicating efforts to increase their productivity.

One of the most heavily loaded operating modes of bucket elevators is the drive start-up mode, during which considerable dynamic loads and shock effects occur, adversely affecting the reliability and durability of the structure. Known technical solutions aimed at reducing dynamic loads, such as the use of frequency converters or fluid couplings, are not always appropriate due to increased structural complexity, higher costs, and elevated operating expenses.

In this regard, the development of passive mechanical systems capable of effectively attenuating start-up loads without significantly complicating the drive design is of particular relevance. One promising approach involves the use of elastic systems with mechanical feedback, which make it possible to form specified elastic characteristics and adapt the dynamic behavior of the drive during transient operating conditions.

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The purpose of this study is to analyze and synthesize bucket elevator drive mounting structures based on graph models using the theory of modified kinematic graphs.

2. Literature analysis

Modern approaches to modeling mechanical systems increasingly rely on graph-based representations, which allow formalizing the structural, kinematic, and dynamic interactions of system components. Graph models can be classified according to the type of information they convey, and each type has been widely used in different areas of mechanical engineering.

For modeling energy interactions within mechanical systems, force graphs are commonly applied. These models have been successfully used by Schneider, Pfeiffer, and Borutsky [1, 2] in studies related to machine dynamics, hydraulics, and electromechanics. Force graphs provide a convenient tool for analyzing the transmission of forces and energy flows between system elements.

Stiffness graphs (or structural graphs) have been employed in the works of Uicker, Pennock, Shigley, and Tsai [3, 4] for modeling elastic systems and building structures. These graphs are particularly useful in evaluating the rigidity and deformation characteristics of mechanical assemblies, enabling the prediction of structural behavior under load.

Another widely used approach involves kinematic graphs, which describe the connectivity and mobility of mechanical systems. Pioneering studies by Jain, and Pennock. [5, 6] applied kinematic graphs to analyze mechanical and robotic systems, determining degrees of freedom, identifying redundant constraints, and synthesizing mechanisms with desired functional properties. Kinematic graphs provide a systematic framework for structural analysis and mechanism design.

Finally, constraint graphs have been utilized to model the dynamics of multi-body systems with interacting components. Jain and Karnopp [5, 7, 8] employed constraint graphs in studies of elastic and dissipative connections, effectively capturing the dynamic behavior of complex assemblies and facilitating the simulation of multi-body interactions.

Overall, the literature indicates that combining different types of graph models allows for a more comprehensive analysis of mechanical systems. By integrating kinematic, force, and stiffness information, hybrid graph models can support the design, optimization, and synthesis of mechanical systems, including drives for bucket elevators, where both structural compactness and dynamic performance are critical.

3. Purpose of research

The aim of the study is to test methods for representing mechanical systems in the form of graphs and to apply these methods for the analysis of existing and the synthesis of new bucket elevator drive designs. The research involves identifying structural and design differences between drives by highlighting the parameters that determine these differences, as well as developing features and criteria that allow an optimal choice among multiple possible solutions, ensuring structural compactness and maximum functional capabilities. In addition, the study focuses on the formulation and evaluation of criteria for selecting the optimal solution, which enables a reasoned determination of the most effective variant among available alternatives. Achieving this objective provides a systematic approach to formalizing the processes of structural analysis and synthesis of bucket elevator drives while taking into account design and technological constraints.

4. Research method

Based on the considered key types of graph models, it can be concluded that their application makes it possible to solve a wide range of engineering problems. Such problems include the structural analysis of mechanisms, in particular the determination of their mobility using the Chebyshev–Grübler–Kutzbach criterion, dynamic modeling of mechanical systems (Lagrange equations, bond graph method), optimization of mechanisms at the stage of their design implementation (elimination of redundant constraints, mass minimization), as well as the kinematic analysis of manipulators in robotic systems.

When designing mechanisms with prescribed functional characteristics, primary attention is paid to the initial structural analysis of existing analogues. This analysis, in turn, requires the use of various graph models, applied both individually and in certain combinations. Despite the diversity of mechanical problems, their relationship with the functional characteristics of a mechanism makes it possible to identify common patterns, which creates the prerequisites for formalizing the processes of analysis and synthesis based on graph theory.

In solving problems of analysis and synthesis in a mechanical formulation, the most commonly used models are kinematic graphs, which represent the structure of a mechanism and define kinematic constraints between its links; force graphs, which are used to model the processes of force transmission between system elements; and functional graphs, which describe the laws of motion transformation. However, the complexity and non-standard nature of modern analysis and synthesis problems in mechanical engineering necessitate the development and application of combined graph models. Such combined models should take into account not only the existing topological constraints of the mechanical system but also ensure the compatibility of functional requirements, along with the possibility of automation and algorithmization of the search for optimal structural solutions.

The studies conducted in this chapter are based on a combined graph model developed by Professor I. I. Sydorenko, which establishes the theoretical foundations and corresponding methodologies for the analysis and synthesis of controlled elastic systems using a combined graph-based approach, referred to by the author as the modified kinematic graph [9, 10]. This theory represents a further development of the theory of analysis and synthesis of mechanisms based on kinematic graphs [11, 12, 13].

The results of studies related to the validation of the above-mentioned theory made it possible to formulate the basic principles of structural synthesis of elastic systems of this type [10 – 14].

According to one of the principles of the applied theory, an idealized representation of the structure of a conventional planar passive device having a single parameter (for example, the stiffness coefficient) subject to controlled variation is a modified kinematic graph (non-bipolar). Such a graph characterizes negative or zero mobility of the considered device, which is determined by the following equation:

$$W = 3\cdot(p_{\Sigma}-1)-2q_5-q_4-q_c \le 0\,, \tag{1}$$

where:

p_{Σ} – the total number of graph vertices corresponding to the number of rigid links of the device;

$q_{(i)}$ – the number of graph edges equal to the number of kinematic pairs of the i-th class;

q_c – the number of graph edges corresponding to the number of non-kinematic connections (elastic, dissipative, etc.) between the rigid links of the device, this component does not affect the kinematic characteristics of the device and can therefore be considered as a virtual kinematic pair of the 4th class.

According to another principle stated in the applied theory, the nature of kinematic control of the properties of a given device, when it is modeled using a modified kinematic graph, can be assessed by an indicator in the form of the cyclomatic number. For controlled elastic mechanisms, this indicator is defined as follows:

$$\sigma = q - p + 1 \ge 2\,, \tag{2}$$

where:

q – the number of graph edges, regardless of their type;

p – the number of graph vertices, also regardless of their type.

Expressions (1) and (2) make it possible to analyze the device under consideration in order to assess the feasibility of controlling its elastic characteristic, and also provide the possibility of synthesizing such a device by adding to the model the required number of structural groups and elastic elements, guided by the specified indicators.

Considering that multiple alternative solutions are possible in the synthesis process, an indicator in the form of the energy of the modified kinematic graph is used to select the optimal synthesis option; for the optimal solution, this indicator should be minimal. The energy of a graph $E(G)$, as a quantity from spectral graph theory, is understood as the sum of the absolute values of the eigenvalues of its adjacency matrix. For a simple graph G with n vertices, its adjacency matrix A is a square matrix of size $n\times n$. Typically, the element A_{ij} is defined as follows: it equals (1) if vertex i is connected by an edge to vertex j, or 0 if vertices i and j are not connected (similarly for $i=j$, since the graph is simple). In the presented methodology, the use of the adjacency matrix is defined with the application of certain weighting coefficients that reflect a particular prevailing factor (from the standpoint of technology, durability, cost, etc.); moreover, the more significant the factor, the lower its weighting coefficient. Based on these conditions, the eigenvalues (roots of the characteristic polynomial) of the adjacency matrix A are determined according to the expression:

$$\varphi(\lambda) = \det(A-\lambda I)\,, \tag{3}$$

where: I – the identity matrix and det – the determinant.

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Considering that for a graph with n vertices n eigenvalues can be obtained: $\lambda_1, \lambda_2, \ldots, \lambda_n$, then the energy of a graph is understood as the sum of the absolute values of all eigenvalues:

$$E(G) = \sum_{1}^{n} |\lambda_i| . \tag{4}$$

Let us consider the above-mentioned theory in application to the subject of this study. For this purpose, an analysis of two designs (Structure A and Structure B) of elastic mechanisms for fastening bucket elevator drives in different directions of force (Fig. 1) was conducted, and the most optimal of them was determined.

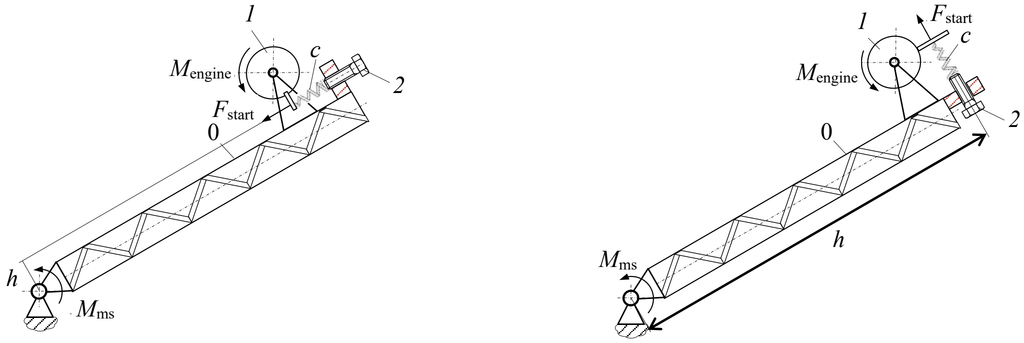

Fig. 1. Kinematic diagrams of the elastic mechanism for mounting bucket elevator drives: *a* – Structure A; *b* – Structure B; M_{engine} – is the engine torque; F_{starting} – is the excitation force caused by the engine start; M_{ms} – is the torque of the metal structure; c – is the spring stiffness; h – is the lever arm; *1* – electric motor; *2* – elastic mechanism

Based on the kinematic schemes (Fig. 1), the corresponding models in the form of modified kinematic graphs (Fig. 2) are tripolar graphs and, therefore, can be analyzed using the methodology presented above.

Fig. 2. Models of the considered structures in the form of modified kinematic graphs: structure A (*a*); structure B (*b*)

The calculation of the degree of mobility according to expression (1) yields the same result in both cases and determines the mobility of the models:

$$W = 3 \cdot (3-1) - 2 \cdot 1 - 1 - 1 = 2 , \tag{5}$$

which, according to the adopted methodology, does not define the considered structures as self-controlled (the condition $W \leq 0$ is not met). However, control of the characteristic q_c is possible with the mutual displacement of the elements of the structures defined by the poles of the graph p_0 and p_2.

The statement about the absence of self-control, obtained in the above calculation, is also confirmed by the calculation of the cyclomatic number according to expression (2):

$$\sigma = 3 - 3 + 1 = 1 , \tag{6}$$

for self-controlled devices, this indicator must satisfy the condition $\sigma \geq 2$.

However, the adopted methodology allows us to choose the optimal solution among several existing ones. To do this, we will compile for the structure under consideration the corresponding adjacency matrices with the introduction of weight coefficients, which take into account both the direction of action of the elastic force relative to the longitudinal axis of the metal structure of the bucket elevator, and the technological features associated with the manufacture of kinematic pairs of the 5-th and 4-th classes. Moreover, the direction of the elastic force along the longitudinal axis of the metal structure will be considered more preferable (the weight coefficient is minimal) than the direction of the elastic force perpendicular to the axis of the metal structure. The adopted values of the weight coefficients in the adjacency matrix of the structures under consideration are given in the corresponding table (Table 1).

Table 1

Determination of weight coefficients in the matrix of structures under consideration

Structure	Edge of a graph (poles)	Weighting factor	Explanation
A	$q_c(p_1–p_2)$	1 (min)	Elastic force along the longitudinal axis of the metal structure
B	$q_c(p_1–p_2)$	2 (max)	Elastic force perpendicular to the longitudinal axis of the metal structure
A – B	$q_5(p_0–p_1)$	1 (min)	Less technologically complex
A – B	$q_4(p_0–p_2)$	2 (max)	More technologically complex

Given that for a graph with three vertices, we can obtain three eigenvalues for the adjacency matrix:

– For structure A we obtained: $\lambda_{1A} = –2$; $\lambda_{2A} = –1.46$; $\lambda_{3A} = 2.73$;

– For structure B we obtained: $\lambda_{1A} = –1$; $\lambda_{2A} = –5.24$; $\lambda_{3A} = 6.24$.

Therefore, the calculated value of the graph energy according to expression (4) for Structure A is E(A) = 6.19, for Structure B it is E(B) = 12.48.

Taking into account the adopted weight coefficients, the adjacency matrices of the structures under consideration have the form (the matrices have the same structure, but different weighting coefficients):

$$\text{For Structure A} = \begin{array}{c|c|c|c} & p_0 & p_1 & p_2 \\ \hline p_0 & 0 & q_5 & q_c \\ \hline p_1 & q_5 & 0 & q_4 \\ \hline p_2 & q_c & q_4 & 0 \end{array} = \begin{array}{c|c|c|c} & p_0 & p_1 & p_2 \\ \hline p_0 & 0 & 1 & 1 \\ \hline p_1 & 1 & 0 & 2 \\ \hline p_2 & 1 & 2 & 0 \end{array} \qquad (7)$$

$$\text{For Structure B} = \begin{array}{c|c|c|c} & p_0 & p_1 & p_2 \\ \hline p_0 & 0 & q_5 & q_c \\ \hline p_1 & q_5 & 0 & q_4 \\ \hline p_2 & q_c & q_4 & 0 \end{array} = \begin{array}{c|c|c|c} & p_0 & p_1 & p_2 \\ \hline p_0 & 0 & 1 & 2 \\ \hline p_1 & 1 & 0 & 2 \\ \hline p_2 & 2 & 2 & 0 \end{array} \qquad (8)$$

Based on the results of the calculations E(A) < E(B), the applied method indicates a more rational Structure A, which is actually confirmed by the analysis in Fig. 2, which shows that the elastic force shoulder $h_A < h_B$, which determines a smaller influence of the moment relative to the attachment point of the metal structure of the bucket elevator.

5. Conclusions

Based on the results of the conducted studies, the following conclusions have been drawn:

1. Despite the effectiveness of standard approaches to applying graph theory for the analysis and synthesis of technical systems, the ambiguity and multi-criteria nature of such problems make the development and implementation of combined graph models preferable. These hybrid structures, by integrating information on kinematics, forces, and functional interactions of the modeled mechanical systems, provide comprehensive system-level modeling and enhance the adequacy of decision-making.

2. The studies have validated the effectiveness of the adopted methodology using modified kinematic graphs for analyzing existing designs and synthesizing new configurations of bucket elevator drives, particularly with respect to optimizing their structure and functional capabilities.

3. As a criterion for selecting the optimal solution from a set of possible alternatives in the synthesis of elevator drives, the successful application of the graph energy $E(G)$, derived from spectral graph theory, was adopted. It was established that the optimal solution corresponds to the minimum

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value of the graph energy. This approach allows formalizing the selection process while taking into account both structural and technological/design factors through weighted adjacency matrices.

4. The validation of the adopted methodology and the corresponding methods of representing a technical system via a modified kinematic graph, along with the availability of an effective criterion for optimal selection, enables the formalization and algorithmization of the structural analysis and synthesis processes of bucket elevator drives. This ensures the selection of solutions that combine structural compactness with maximal functional capabilities while considering design and technological constraints.

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Курган Володимир Олегович; Volodymyr Kurhan, ORCID: https://orcid.org/0009-0003-9816-5419

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